Vector calculus MA3VC 2016–17: Assignment 1

MA3VC: Part 3 students only.
Handed out: Thursday 20th October. Due: **Thursday 27th October, 12 noon**.
You can use formulas and identities from the lecture notes. Do not use red pen nor pencil.
Marking will be anonymous, so please write your name only on the "assessed work coversheet" and not on your work. Write your student number both on the back of the coversheet and each page of your work.
Total marks: 25. (10% of the total marks for the module.)

(Exercise 1 — 7 marks)
Prove that if f is a (smooth) scalar field and $\vec{\mathbf{G}}$ is an irrotational vector field, then

$$(\vec{\nabla} f \times \vec{\mathbf{G}}) f$$

is solenoidal.

Hint: Do NOT expand in coordinates and partial derivatives. Use instead the vector differential identities of §1.4 and the properties of the vector product from §1.1.2 (recall in particular Exercise 1.15).

(Exercise 2 — 5 marks)
Demonstrate that the field $(\vec{\nabla} f \times \vec{\mathbf{G}}) f$ is indeed solenoidal for $f = xyz$ and $\vec{\mathbf{G}} = \vec{\mathbf{r}}$.

(Exercise 3 — 13 marks)
Consider the vector field $\vec{\mathbf{F}} = x^2 y(\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{k}}) - xy^2 \hat{\boldsymbol{\jmath}}$.

1. (2 marks) Show that $\vec{\mathbf{F}}$ is not conservative.
   Hint: Use the relation between conservative and irrotational fields.
2. (3 marks) Find a vector potential $\vec{\mathbf{A}}$ for $\vec{\mathbf{F}}$.
   Hint: Look for a vector potential that depends on two coordinate variables only.
3. (2 marks) Let $\vec{\mathbf{B}} \neq \vec{\mathbf{A}}$ be any other vector potential of $\vec{\mathbf{F}}$. Prove that the difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is irrotational.
4. (4 marks) Find a vector field $\vec{\mathbf{B}}$ that is simultaneously (i) a vector potential for $\vec{\mathbf{F}}$ and (ii) solenoidal.
   Hint: Given $\vec{\mathbf{A}}$ from the previous question, find a scalar field α such that $\vec{\mathbf{B}} = \vec{\mathbf{A}} + \vec{\nabla}\alpha$ satisfies the request.
5. (2 marks) Can you find a field $\vec{\mathbf{G}}$ that admits $\vec{\mathbf{F}}$ as vector potential? How many such fields exist?

Please check carefully the list of common errors on page 110 of the notes and try not to commit them!
Recall also that the nabla symbol "$\vec{\nabla}$" is not a vector and cannot be treated as such.