

RECONSTRUCTING EARLY STAGES OF PROSTATE CANCER GROWTH: MATHEMATICAL ANALYSIS

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The availability of cancer measurements over time enables the personalized assessment of tumour growth and therapeutic response dynamics. However, many tumours are treated right after diagnosis without collecting longitudinal data, and cancer monitoring protocols may include infrequent measurements. To facilitate the estimation of disease dynamics and better guide ensuing clinical decisions, we investigate an inverse problem enabling the reconstruction of earlier tumour stages by using a single spatial tumour dataset and a biomathematical model describing disease dynamics. We focus on prostate cancer, since aggressive cases are usually treated after a single diagnostic MRI scan. We describe tumour dynamics with an Allen-Cahn phase-field model driven by a generic nutrient that follows reaction-diffusion dynamics. The model is completed with another reaction-diffusion equation for the local production of prostate-specific antigen, which is a key prostate cancer biomarker. We first improve previous well-posedness results by further showing that the solution operator is continuously Fréchet differentiable. We then analyse the backward inverse problem concerning the reconstruction of earlier tumour stages starting from measurements of the model variables at the final time. Since this problem is severely ill-posed, only very weak conditional stability of logarithmic type can be recovered from the terminal data. Nevertheless, by restricting the unknowns to a compact subset of a finite-dimensional subspace, we can further derive an optimal Lipschitz stability estimate. Such results then lead to the development of a locally convergent iterative reconstruction algorithm based on the Landweber scheme. We finally show some numerical experiments validating the obtained theoretical results. This is a joint work with E. Beretta, C. Cavaterra, G. Lorenzo and E. Rocca.