HYBRID TOMOGRAPHY AND NON-VANISHING JACOBIAN PROBLEM

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Parameters reconstruction problem is a kind of inverse problem of the second order partial differential equations. Usually, people use a method called Hybrid tomography to solve this kind of question that is reconstructing parameters by using several solutions which satisfy certain constraints. We are interested in finding which constraint is needed as well as finding a group of candidate solutions (solutions which satisfy such constraint). In fact, for the elliptic operator $Lu = -\text{div}\,(A\nabla u)$, the associated constraint becomes a so called non vanishing Jacobian constraint. Finding a group of solutions which satisfy the non vanishing Jacobian constraint is called the non vanishing Jacobian problem.

In 2020, Giovanni S. Alberti and Yves Capdeboscq considered this non vanishing Jacobian problem in [Alberti, Giovanni S., and Yves Capdeboscq. International Mathematics Research Notices 2022.6 (2022): 4387-4406.] for the general elliptic operator. They enforced non-vanishing constraints for solutions to a second order elliptic partial differential equation by appropriate choices of boundary conditions. They showed that, in dimension $\mathbb{R}^d$, under suitable uniform regularity assumptions, the family of $2d$ solutions such that their Jacobian has maximal rank in the domain is both open and dense. The approach is based on the combination of the Runge approximation property and a Whitney projection argument.

In applications, piece-wise regular (or piece-wise constant) coefficients often appear, especially when reconstructing conductivity in composite media. Classical Runge approximation property results and the Whitney projection argument cannot be used in this case due to the discontinuity of coefficients between different sub-domains.

In this talk, I will introduce how we deal with this piece-wise regular coefficients case. We formulated a new approximation method combining a small dilation discussion and an extended Runge property. We also derived a modified Whitney argument, by constructing a group of vector fields behaving like normal and tangent vectors on each inner boundary. We showed that under suitable piece-wise regularity assumptions, only when $d = 2, 4, 8$, the family of $2d + 1$ solutions such that their Jacobian has maximal rank in the domain is both open and dense. In other dimension, one more solution should be added to the family (in other words, the family of $2d + 2$ solutions) to realize the openness and the density.