STABILIZATION OF A STERILE INSECT TECHNIQUE MODEL BY FEEDBACK LAWS

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The Sterile Insect Technique (SIT) is one of the most ecological methods for controlling insect pests that are responsible for worldwide crop destruction and disease transmission. This technique consists in releasing sterile males into the insect pest population. This approach aims to reduce fertility and, consequently, the target insect population after a few generations. Classical SIT has been modeled and studied theoretically in a large number of papers to derive results to study the success of these strategies using discrete, continuous, or hybrid modeling approaches (recent papers [2, 5, 6]). Despite this extensive research, little has been done concerning the stabilization of the target population near extinction after the decay caused by the massive initial SIT intervention. In this work, we study the global stabilization of a pest population at extinction equilibrium by the SIT method and construct explicit feedback laws that stabilize the model. However, the practical implementation of these feedback controls is limited by the need for continuous and often difficult measurements. Finding a feedback control that ensures the global stability of the system with only more accessible measurements is a complicated mathematical problem. To overcome this, our approach focuses on the one hand, in building an observer for the SIT model to estimate the different states and on the other hand using deep reinforcement learning (RL) to suggest and construct feedback laws that only depend on these measurements (namely the adult mosquito population, which can be measured using pheromone traps).

This is a joint work with Jean-Michel Coron, Luis Almeida, Amaury Hayat and Nathan Lichtlé [1, 3, 4].

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SYMMETRIC APPROXIMATIONS TO NONLINEAR DISPERSIVE EQUATIONS IN NON-SMOOTH REGIMES

YVONNE ALAMA BRONSARD

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This talk deals with the numerical approximation to nonlinear dispersive equations, such as the prototypical nonlinear Schrödinger equation. We introduce novel integration techniques allowing for the construction of schemes which perform well both in smooth and non-smooth settings. We obtain symmetric low-regularity schemes with very good structure preserving properties over long times.

Higher order extensions will be presented, following new techniques based on decorated trees series inspired by singular stochastic PDEs via the theory of regularity structures.

ON THE SHAPE OF SMALL LIQUID DROPS MINIMIZING NONLOCAL ENERGIES

KONSTANTINOS BESSAS

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We study the equilibrium shape of liquid drops minimizing the fractional perimeter under the action of a potential energy. We prove, with a quantitative estimate, that the small volume minimizers are convex and uniformly close to a ball. This is a joint work with Matteo Novaga (Pisa) and Fumihiko Onoue (München).

TRANSFER OF CERCIGNANI'S CONJECTURE-TYPE INEQUALITIES FROM THE CLASSICAL TO THE FERMIONIC BOLTZMANN EQUATION AND AN APPLICATION

THOMAS BORSONI

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The fermionic Boltzmann (Boltzmann-Fermi-Dirac or fermionic Nordheim) equation is a kinetic description of rarefied gases of fermions (e.g. electrons). The setting is similar to the classical Boltzmann equation, with a modification of the collision operator, in order to take into account the Pauli exclusion principle. As a result, the corresponding equilibrium distributions (Fermi distributions) and the relevant entropy (Fermi entropy) do also differ from their classical analogues (Maxwellian distribution and Boltzmann entropy).

Entropy methods are a at the core of quantitative studies on relaxation to equilibrium. For the classical Boltzmann equation, the quantitative decay of the relative entropy to equilibrium is provided by a relationship between the relative entropy to equilibrium and its dissipation in time. These relationships are called «Cercignani's conjecture-type» inequalities.

In this talk, I present a method of «transfer» of inequalities, which establishes an (almost) equivalence, in terms of entropy inequalities, between the classical and the fermionic Boltzmann cases, hence providing a large class of such results for solutions to the fermionic Boltzmann equation, and therefore, quantitative rates of convergence towards equilibrium. I present an application of this result, in which explicit polynomial convergence to equilibrium is rigorously obtained.

FROM LIONS' INEQUALITY TO CONSTRUCTIVE CONVERGENCE RATES OF KINETIC EQUATIONS

GIOVANNI BRIGATI

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We start from an inequality by J.-L. Lions, dating back to the '60s, which compares a norm of a function to a weak norm of the gradient of the function itself. Such result is of broad interest, ranging from mathematical elasticity to PDEs, and numerics. More recently, Lions' inequality has been playing a crucial role in determining the convergence rates to equilibrium in dissipative kinetic equations. In this talk, we show new Lions' inequalities in weighted Sobolev spaces, with constants being explicit in the data of the problem, and in the dimension of the space. A novel weak Lions' inequality is also introduced, as a tool to overcome "weak confinement" in the kinetic setting.

TOWARDS OPERATOR LEARNING FOR IONIC MODELS IN BIOMATHEMATICS

EDOARDO CENTOFANTI

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Ionic models are among the most extensively studied dynamical systems in biomathematics, as they play a crucial role in modelling electrophysiology at the cellular scale. In particular, they are a crucial component in cardiac modelling since they account for the excitability of the cellular membrane and they are responsible for the action potential. Since they may significantly contribute to the computational complexity of the problem, it is important to develop alternative techniques that minimize their impact on the global solution time. In this talk, we will introduce a strategy for exploiting Operator Learning techniques, such as DeepONet, Fourier Neural Operator (FNO) and Wavelet Neural Operator (WNO), to solve these systems more efficiently. Specifically, we will compare the accuracy of the trained models with the ones solved numerically and discuss the capabilities of these architectures in reconstructing the desired dynamics.

MECHANICALLY CONSISTENT MODELING OF FLUID-STRUCTURE-CONTACT INTERACTION WITHOUT COLLISION PARADOX

MARGUERITE CHAMPION

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Joint work with Miguel Fernandez, Céline Grandmont and Fabien Vergnet.

The numerical simulation of systems involving fluid-structure-contact interaction raises many modeling, mathematical and numerical issues. It is also crucial for numerous biomedical applications (e.g., native or artificial cardiac valves). Modeling contact between solids adds challenging difficulties to fluid-structure interaction (FSI). First, in some configurations, FSI models are unable to predict contact; this is the so called no collision paradox. A second major issue is to obtain mechanically consistant models. Indeed, the simple addition of a contact constraint leads to mechanical inconsistencies like unphysical void creation at releases from contact or unbalanced stress at contact. A favored approach is to consider a porous modeling of the fluid seepage induced by the roughness of the contacting solid. In this talk we will show that, in the case of a rigid disk moving over a fixed horizontal plane, adding a surfaced Darcy model on the plane does encompass contact, and hence removes the above mentioned non collision paradox of traditional FSI models which rely on Dirichlet or Dirichlet/Navier boundary conditions. Numerical evidence on this result will also be provided. Finally, we will explore the extension of poroelastic modeling of seepage to the case of moving elastic solids.

MODEL ORDER REDUCTION IN SUPPORT OF THE VIRTUAL ELEMENT METHOD

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Virtual elements (VEM) are a family of numerical methods for the approximation of PDEs which can easily handle complex geometries and their discretization by means of arbitrarily-shaped polygons/polyhedra. Since the basis functions of the VEM space are themselves solutions of PDEs, their explicit evaluation is not required, and several quantities are not computed exactly. For instance, the nonpolynomial contribution is handled by a stabilization term when constructing the bilinear form and neglected when computing the error.

We propose a Reduced Basis (RB) approach for efficiently solving the equation associated to each virtual basis function. The idea is to replace the stabilization term with an actual approximation of the nonpolynomial contribution. We show that this operation produces good results even if done in a very rough way. This novel approach improves the convergence properties of VEM when applied, for instance, to anisotropic second order diffusion equations, when standard stabilization recipes show poor performance. In post-processing framework, it is well known that, when a PDE is solved with VEM, the degrees of freedom of the discrete solution allow only the computation of projections onto discontinuous polynomial spaces, so that the solution is not conforming. The RB approximation of the virtual functions can also be exploited for reconstructing conforming solution in the VEM space. This task can be useful to carry out operations such as visualization, reconstruction in subdomains, pointwise evaluation and evaluation of the conforming error when benchmarking the method.

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HYBRID TOMOGRAPHY AND NON-VANISHING JACOBIAN PROBLEM

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Parameters reconstruction problem is a kind of inverse problem of the second order partial differential equations. Usually, people use a method called Hybrid tomography to solve this kind of question that is reconstructing parameters by using several solutions which satisfy certain constraints. We are interested in finding which constraint is needed as well as finding a group of candidate solutions (solutions which satisfy such constraint). In fact, for the elliptic operator $Lu = -\text{div}\,(A\nabla u)$, the associated constraint becomes a so called non vanishing Jacobian constraint. Finding a group of solutions which satisfy the non vanishing Jacobian constraint is called the non vanishing Jacobian problem.

In 2020, Giovanni S. Alberti and Yves Capdeboscq considered this non vanishing Jacobian problem in [Alberti, Giovanni S., and Yves Capdeboscq. International Mathematics Research Notices 2022.6 (2022): 4387-4406.] for the general elliptic operator. They enforced non-vanishing constraints for solutions to a second order elliptic partial differential equation by appropriate choices of boundary conditions. They showed that, in dimension $\mathbb{R}^d$, under suitable uniform regularity assumptions, the family of $2d$ solutions such that their Jacobian has maximal rank in the domain is both open and dense. The approach is based on the combination of the Runge approximation property and a Whitney projection argument.

In applications, piece-wise regular (or piece-wise constant) coefficients often appear, especially when reconstructing conductivity in composite media. Classical Runge approximation property results and the Whitney projection argument cannot be used in this case due to the discontinuity of coefficients between different sub-domains.

In this talk, I will introduce how we deal with this piece-wise regular coefficients case. We formulated a new approximation method combining a small dilation discussion and an extended Runge property. We also derived a modified Whitney argument, by constructing a group of vector fields behaving like normal and tangent vectors on each inner boundary. We showed that under suitable piece-wise regularity assumptions, only when $d = 2, 4, 8$, the family of $2d + 1$ solutions such that their Jacobian has maximal rank in the domain is both open and dense. In other dimension, one more solution should be added to the family (in other words, the family of $2d + 2$ solutions) to realize the openness and the density.

PRESSURE JUMP IN THE CAHN-HILLIARD EQUATION

CHARLES ELBAR

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We model a tumor as an incompressible flow considering two antagonistic effects: repulsion of cells when the tumor grows (they push each other when they divide) and cell-cell adhesion which creates surface tension. To take into account these two effects, we use a 4th-order parabolic equation: the Cahn-Hilliard equation. The combination of these two effects creates a discontinuity at the boundary of the tumor that we call the pressure jump. To compute this pressure jump, we include an external force and consider stationary radial solutions of the Cahn-Hilliard equation. We also characterize completely the stationary solutions in the incompressible case, prove the incompressible limit and prove convergence of the parabolic problems to stationary states.

RECONSTRUCTING EARLY STAGES OF PROSTATE CANCER GROWTH: MATHEMATICAL ANALYSIS

MATTEO FORNONI

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The availability of cancer measurements over time enables the personalized assessment of tumour growth and therapeutic response dynamics. However, many tumours are treated right after diagnosis without collecting longitudinal data, and cancer monitoring protocols may include infrequent measurements. To facilitate the estimation of disease dynamics and better guide ensuing clinical decisions, we investigate an inverse problem enabling the reconstruction of earlier tumour stages by using a single spatial tumour dataset and a biomathematical model describing disease dynamics. We focus on prostate cancer, since aggressive cases are usually treated after a single diagnostic MRI scan. We describe tumour dynamics with an Allen-Cahn phase-field model driven by a generic nutrient that follows reaction-diffusion dynamics. The model is completed with another reaction-diffusion equation for the local production of prostate-specific antigen, which is a key prostate cancer biomarker. We first improve previous well-posedness results by further showing that the solution operator is continuously Fréchet differentiable. We then analyse the backward inverse problem concerning the reconstruction of earlier tumour stages starting from measurements of the model variables at the final time. Since this problem is severely ill-posed, only very weak conditional stability of logarithmic type can be recovered from the terminal data. Nevertheless, by restricting the unknowns to a compact subset of a finite-dimensional subspace, we can further derive an optimal Lipschitz stability estimate. Such results then lead to the development of a locally convergent iterative reconstruction algorithm based on the Landweber scheme. We finally show some numerical experiments validating the obtained theoretical results. This is a joint work with E. Beretta, C. Cavaterra, G. Lorenzo and E. Rocca.

INVESTIGATION OF THE MULTIPHYSICS FLOW DYNAMICS OF THE CEREBROSPINAL FLUID IN THE HUMAN BRAIN BY A POLYTOPAL METHOD

IVAN FUMAGALLI

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In physiological conditions, the Cerebrospinal Fluid (CSF) plays a crucial role in washing out misfolded proteins from the brain, by filtrating through the cerebral tissues and flowing in its hollow cavities. Indeed, the impairment of this waste clearance mechanism is strongly associated with the onset and development of many neurodegenerative diseases. CSF filtration in the cerebral tissue is also in strict connection with blood perfusion of the brain and it can be modeled by Multiple-network Poro-Elasticity (MPE) equations, while its flow in the brain ventricles can be described by Stokes equations. The coupled Stokes-MPE model is discretized by a Polytopal Discontinuous Galerkin (PolyDG) method, which is particularly suitable to efficiently deal with the brain's complex geometry, and for which rigorous stability and convergence results are provided [1]. Its implementation in the PolyDG library lymph [2] (https://lymph.bitbucket.io/) allows for verification tests and simulations in realistic 2D geometries, while fully 3D computational models in patient-specific geometries are developed based on FEniCS (https://fenicsproject.org/).

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A PÉCLET-ROBUST DISCONTINUOUS GALERKIN METHOD FOR NONLINEAR DIFFUSION WITH ADVECTION

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Joint work with L. Beirao da Veiga (Università di Milano-Bicocca) and D.A. Di Pietro (Université de Montpellier).

Discontinuous Galerkin (DG) methods were introduced in the 70s [1] and they are nowadays widely regarded as the reference methods for advection-dominated problems. When a polynomial degree $k \geq 1$ is used, classical error estimates for linear diffusion-advection(-reaction) problems show that the error contribution stemming from diffusive terms is $\mathcal{O}(h^k)$ (with h denoting the meshsize), while the one stemming from advective terms is $\mathcal{O}(h^{k+\frac{1}{2}})$.

The present talk, based on [2], aims to show new Péclet-dependent error estimates for a problem with linear advection-reaction and nonlinear p-type diffusion, with Sobolev indices $p \in (1, \infty)$. Convergence analyses for various DG schemes applied to pure p-type diffusion setting can be found, e.g., in [3, 4]. To the authors' knowledge, an investigation of model problems including both advection and nonlinear diffusion is missing in the literature of DG elements. Especially if one aims at developing sharp estimates which respect the local nature of diffusion and convection, the interaction between the (linear) advection and the nonlinear diffusion cannot be accounted for through a simple combination of known techniques, as the estimate of each term becomes dependent on the local regime.

The discretization of the nonlinear diffusion term is based on the full gradient including jump liftings and interior-penalty stabilization while, for the advective contribution, we consider a strengthened version of the classical upwind scheme. The peculiarity of our error estimates is that they track the dependence of the local contributions to the error on local Péclet numbers. In the linear case, corresponding to $p = 2$, local Péclet numbers can be computed based on the sole knowledge of the problem data and the mesh, making it possible to identify a priori advection- and diffusion-dominated elements/faces. We emphasize that our results hold for general polygonal and polyhedral meshes, which we believe is an important asset. The present contribution furthermore sets the stage for future publications developing pressure robust and advection-robust finite elements for time-dependent Navier–Stokes type equations modeling incompressible fluid flows with non-Newtonian rheology.

In the present talk, after presenting the model and the numerical method, we will outline the theoretical results. Finally, a set of numerical tests supporting the theory will be shown.

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GLOBAL PROPAGATION OF ANALYTICITY AND UNIQUE CONTINUATION FOR SEMILINEAR WAVES

CRISTOBAL LOYOLA

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In this talk, we explore the global propagation of analyticity and unique continuation for solutions of the semilinear wave equation, where the nonlinearity is assumed to be subcritical, defocusing, and analytic. We will first discuss how an analytic in time regularization can be obtained in a finite-time setting for solutions who are observed to be zero in a small subset of the domain. Central to our approach is the assumption that the observation zone satisfies the geometric control condition (GCC). This will allow us to employ tools coming from the context of control theory to achieve the result. As a consequence, we are able to obtain propagation of analyticity and a unique continuation property, both in finite time under the GCC. Finally, we will stress that the analytic regularization property in finite-time was actually obtained in an abstract framework, allowing to explore similar results for other PDEs. These results are part of a joint work with C. Laurent (LJLL).

INTEGRAL EQUATION METHODS FOR ACOUSTIC SCATTERING BY FRACTALS

ANDREA MOIOLA

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We study sound-soft time-harmonic acoustic scattering by general scatterers, including fractal scatterers, in 2D and 3D space. For an arbitrary compact scatterer we reformulate the Dirichlet boundary value problem for the Helmholtz equation as a first-kind integral equation (IE) involving the Newton potential. The IE is well-posed, except possibly at a countable set of frequencies, and reduces to existing single-layer boundary IEs when the scatterer is the boundary of a bounded Lipschitz open set, a screen, or a multi-screen. When the scatterer is uniformly of d-dimensional Hausdorff dimension in a sense we make precise (a d-set), the operator in our equation is an integral operator with respect to the d-dimensional Hausdorff measure, with kernel the Helmholtz fundamental solution, and we propose a piecewise-constant Galerkin discretization of the IE, which converges in the limit of vanishing mesh width. When the scatterer is the fractal attractor of an iterated function system of contracting similarities, we prove convergence rates, and describe a fully discrete implementation including quadrature rules for singular integrals on fractals. We present numerical results for a range of examples; our software is available as a Julia code.

This is a joint work with A. Caetano (Aveiro), S.N. Chandler-Wilde (Reading), X. Claeys (LJLL), A. Gibbs (UCL), D.P. Hewett (UCL). Details can be found in the preprint arXiv:2309.02184.

LOW-RANK METHODS AND SOLVERS FOR ISOGEOMETRIC ANALYSIS

MONICA MONTARDINI

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Similarly to other numerical methods to solve Partial Differential Equations (PDEs), computing the solution of PDEs with Isogeometric Analysis suffers from the so-called curse of dimensionality, i.e. memory storage and computational effort grow exponentially with respect to the dimension of the problem. In this talk we propose low-rank techniques that can overcome this issue with the aim of computing the solution with roughly $O(n)$ FLOPs, where n is the number of univariate degrees of freedom [1]. A low-rank decomposition of the linear system matrix kernel is combined with a new suited iterative solver. In particular, the non-tensor product coefficients are approximated with the sum of few Kronecker-product functions, and thus the linear system matrix results in the sum of few Kronecker-product matrices. This yields a small memory footprint and cost for the matrix products. The techniques to approximate the linear system matrix in low-rank format are already present in literature. The novelty of our work is the development of a specialized iterative solver combined with a preconditioning strategy. This low-rank technique and the preconditioning strategy can be extended to multipatch geometries, using an overlapping Schwarz method where the subdomains can be defined as unions of neighbouring patches [2]. Finally, we also show some numerical experiments.

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SPREADING FOR A REACTION-DIFFUSION SYSTEM WITH FORCED SPEED

NGA NGUYEN

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In this work, we study a compartmental model using a partially degenerate reaction-diffusion system to describe the dynamics of the mosquito population under biological control with the Sterile Insect Technique. This technique consists of mass releases of sterilized male mosquitoes that mate with indigenous females yet produce no fertile offspring. Our model considers a release in a 'forced' moving interval in the opposite direction to the natural propagation. We prove that for a sufficiently large number of sterile males released, the population spreads at the same speed as the release and the zero state invades the persistent state, which means we successfully reverse mosquito propagation. The proofs rely on constructing generalized sub- and super-solutions by gluing elementary functions suitably. In the one-dimensional case, we study the monostable system in which the population initiating from any non-trivial, compactly supported initial data persists and spreads in the whole space. We design an exponentially decreasing release function to avoid the reinvasion phenomenon in this monostable case. The results are generalized to the 2D problem for both bistable and monostable systems.

PRESERVATION OF FUNCTIONAL INEQUALITIES UNDER LOG-LIPSCHITZ PERTURBATIONS

PABLO LOPEZ RIVERA

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Given a probability measure satisfying some functional inequalities (Poincaré, log-Sobolev, etc.), it is natural to wonder if these remain valid for a perturbation of the measure. In particular, if there exists a globally Lipschitz map pushing forward the source measure towards its perturbation, then it is easy to transport certain functional inequalities. For example, Caffarelli's contraction theorem states that the optimal transport map between the Gaussian measure and a log-concave perturbation is 1-Lipschitz.

In this talk I will show how such a map exists if we consider log-Lipschitz perturbations of a measure on a Riemannian manifold, via the interpolation given by the Langevin diffusion associated to the source measure (aka Kim-Milman's heat flow transport map), assuming as well control on the curvature of the manifold at first and second order in the sense of Bakry-Émery-Ricci.

RESONANCES AS A COMPUTATIONAL TOOL

KATHARINA SCHRATZ

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A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

WASSERSTEIN SOBOLEV SPACES AND APPLICATIONS TO THE COMPUTATION OF THE WASSERSTEIN DISTANCE

GIACOMO SODINI

Universität Wien, Vienna, Austria

We study the metric Sobolev space on the space of Borel probability measures on the Euclidean space endowed with the Wasserstein 2-distance, proving its Hilbertianity and the density in energy of cylindrical functions. We then provide applications of the abstract density result to the efficient computation of the Wasserstein distance. The talk is based on joint works with Pascal Heid (TU Munich), Massimo Fornasier (TU Munich) and Giuseppe Savaré (Bocconi University, Milano).

HIGH ORDER RECOVERY OF GEOMETRIC INTERFACES FROM CELL-AVERAGE DATA

AGUSTIN SOMACAL

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In image processing edge-adapted methods are used to reconstruct high-resolution images from coarser cell averages. When images are piecewise smooth functions, interfaces can be approximated by a pre-specified functional class through optimization LVIRA or specific pre-processing ENO-EA. First we present a framework to analyze the reconstruction capabilities of non-linear families and use it to prove that LVIRA is a second order method. Then we show how to build fast higher order methods to reconstruct interfaces as well as two strategies to deal with non-smooth interfaces presenting corners.

OPTIMIZATION OF POLYTOPAL MESHES FOR THE VIRTUAL ELEMENT METHOD

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We introduce an optimization procedure designed for generic polygonal or polyhedral meshes, specifically tailored for the Virtual Element Method (VEM). Upon evaluating the local quality of mesh elements using a VEM-specific quality indicator [1, 2], clusters of elements are agglomerated to optimize the global mesh quality. As a result, the discretization becomes significantly lighter, allowing for the removal of up to 80At the same time, the price to pay in terms of accuracy is negligible: the convergence rate of the VEM is maintained in optimized meshes, and the approximation errors remain comparable to those of the original meshes. Notably, the optimization process acts as a regularizer for low-quality meshes by eliminating the most pathological elements. This regularization effect becomes evident in cases where the original meshes cause VEM divergence, whereas the optimized meshes lead to convergence. We present applications of this approach to different types of planar and volumetric discretizations, ranging from classical simplicial meshes and grids to non-convex and low-quality polytopal meshes. We also show how the procedure can be effectively applied in the context of Discrete Fracture Networks [3] and in the simulation of time-dependent problems on real CAD models.

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