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Urne di composizione incognita e paradossi quantistici

Modena
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U é una scatola con 4 palline,

N N B ?

2 nere, 1 bianca, e l'altra inserita (senza guardarne il colore) dopo averla estratta a caso da un'altra scatola contenente un ugual numero di palline bianche e nere.

Quindi, posto

$I_B =$ la pallina inserita é bianca ,

$I_N =$ la pallina inserita é nera ,

si ha

$$P(I_B) = P(I_N) = \frac{1}{2}. \tag{1}$$

Ovviamente, I_B ,I_N sono eventi incerti, e NON fatti osservati. Se si estrae una pallina a da **U**, qual é la probabilitá dell'evento

$$\text{"}A = a \text{ é bianca" ?}$$

Una risposta immediata si ha per le probabilitá **condizionate**

$$P(A|I_B) = \frac{1}{2} \quad , P(A|I_N) = \frac{1}{4} . \tag{2}$$

D'altra parte,

$$P(A) = P(I_B)P(A|I_B) + P(I_N)P(A|I_N) = \tag{3}$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4} = \frac{3}{8} .$$

Can this value be **really** equal to $P(A)$? This assessment of $P(A)$ depends on the circumstance that the colour of the inserted ball is not known. So $P(A)$ is in...our mind (it is "epistemic"). But an "experimental measure" of $P(A)$ by a large number of drawings (with replacement) from the box **U**, leads to a **fact**, i.e. a "long run" observed frequency that is almost certainly equal either to $1/2$ or to $1/4$, and **not** to $3/8$.

Notice that the previous frequentistic evaluation of the probability of the event A is in fact an evaluation of the *conditional* probabilities $P(A|I_H)$ corresponding to the *given but unknown* H (whose possible values are B and N): so "in the long run" the *observed* frequency takes (approximately) one of the two values (2).

Instead the value (3), which is a perfectly legitimate and sensible evaluation of the probability of A, should be regarded as unacceptable from the point of view of a *strict* frequency interpretation of probability.

Interpreting A as $A|\Omega$ and I_H as $I_H|\Omega$, it is easily seen that the value $P(A)$ given by (3) **is** a coherent extension of the conditional probabilities $P(A|I_H)$ and $P(I_H|\Omega)$, while in general a value of $P(A)$ obtained by measuring a relevant frequency **may not**.

In conclusion, the (putative) lack of validity of formula (3) depends on the identification of the probabilities $P(A|I_H)$ with observed frequencies (corresponding to different and incompatible experiments), each one referring to a different (ideal) box.

Nevertheless these experiments are not (so to say) “mentally” incompatible if we argue in terms of the general interpretation of probability (that is, $P(A|I_H)$ is the degree of belief in A *under the assumption* – not necessarily an observation – “I_H is true”):

then, for a coherent evaluation of $P(A)$ *we must* necessarily rely *only* on the above value obtained by resorting to the second member of (3), even if such probability ... "does not exist" (in the sense that it does not express any sort of "physical property" of the given box).

So it is not surprising that also in quantum mechanical experiments the identification of (conditional) probabilities with some statistical data (observed frequencies) may lead to results which are in contradiction with other experiments, still involving frequencies. (Examples: Bell inequality, the two-slit experiment, Einstein paradox)

The classical two-slit experiment is an interesting illustration of the quantum mechanical way of computing the relevant probabilities. A source emits "identically prepared" particles toward a screen with two narrow openings, denoted S_1 and S_2 . Behind the screen there is a film which registers the relative frequency of particles hitting a small given region A of the film.

Measurements are performed in three different physical situations: both slits open, only slit S_1 open, only slit S_2 open. Let us introduce, for a given particle, the following event, denoted (by abusing notation) by the same symbol of the corresponding physical device:

$$A = \text{the particle reaches the region } A\,,$$

and, for $i = 1, 2,$:

$$S_i = \text{the particle goes through slit } S_i\,.$$

The aforementioned experimentally measured frequencies are usually identified, respectively, with the probabilities $P(A)$, $P(A|S_1)$ and $P(A|S_2)$. Repeated experiments can be performed letting a particle start from the source, and then measuring its final position on the film, to determine whether it is in the region A or not;

moreover we could "measure" the values $P(A|S_1)$ or $P(A|S_2)$ letting be put in function an experimental device allowing the particle going to hit the region A only through the slit S_1 or only through the slit S_2. The latter corresponding frequencies (of going through the relevant slit) are also identified with the probabilities $P(S_1)$ and $P(S_2)$.

Now, irrespective of whether the device has been activated or not, we may obviously write eq.(3) (with S_1 and S_2 in place of I_B and I_N, respectively), since this is a "theorem" of probability. On the other hand, physical experiments give an inequality between left and right hand side of (3), **but this circumstance cannot be used to "falsify" anything, since it refers in fact only to observed frequencies**.

Considering this (putative) paradox as strictly peculiar to quantum physics leads to look on it as an anomaly where the so-called either “particle interpretation” or “wave interpretation” is adopted as more appropriate, depending on relevant specific aspects.

E il passaggio dalla prima alla seconda interpretazione viene detto "collasso" della funzione d'onda su un singolo valore. Ma la funzione d'onda ha lo stesso ruolo della distribuzione di probabilitá sui due eventi incerti I_B ,I_N (non si conosce il colore della quarta pallina), e quindi non é una ..."realtá" verificabile, ma uno strumento formale introdotto per fare previsioni su quello che si potrá osservare in corrispondenza a specifiche condizioni.

Nella visione tradizionale della meccanica quantistica, un oggetto come un elettrone é rappresentato dalla sua funzione d'onda (ed infatti i problemi si presentano solo quando i fisici si comportano come se la funzione d'onda fosse reale ...). Quando si effettua una misurazione, si trova l'elettrone in una data posizione, cioé la funzione d'onda "collassa" su un singolo valore.

Non c'é paradosso: il collasso della funzione d'onda corrisponde semplicemente al fatto che un osservatore rivede la sua assegnazione di probabilitá in base a nuove informazioni.

Il sistema quantistico NON é cambiato in modo strano e inspiegabile, il cambiamento é della funzione d'onda, che..." non esiste" !

Insomma, quello che potrebbe apparire paradossale come *stato ontologico* (cioé della realtá), non lo é come *stato epistemologico* (cioé della conoscenza). Nel caso dell'urna, la misurazione effettiva (frequentista) di $P(A)$ é l'analogo del "collasso" della funzione d'onda su un singolo valore (la quarta pallina, dopo estratta, corrisponde all'osservazione di un elettrone, mentre prima dell'estrazione, come giá osservato, si conosce solo la sua distribuzione di probabilitá).

In the example of the box there is no quantum effect to which the lack of validity of formula (3) could be ascribed. Making computations by the frequency relevant to a given experiment involves (so to say) a choice of a conditional probability, in the sense that it is no more allowed to consider – in the same framework – **other** experiments.

Otherwise the rules of conditional probability (in particular, those referring to a "variable" **conditioning** event) could be violated. In other words: while a convex combination of conditional *probabilities* can be – see formula (3) – a *probability*, a convex combination of conditional *frequencies* in general is not a frequency apt to evaluate a particular probability ...

Concludo con una citazione da

"Bruno de Finetti: PROBABILITA' E INDUZIONE, a cura di P. Monari e D. Cocchi, Ed.CLUEB, Bologna, 1993", pp.267, 269 :

If quantum mechanics is to be viewed as a hypothesis ... it is not directly about **possible states of nature**, *but as a hypothesis about* **states of mind about possible states of nature** *... a framework (within which we formulate hypotheses about particular physical systems) that is not true or false, but useful or not.*

Riferimento bibliografico

R. Scozzafava (1991), "A classical analogue of the two-slit model of quantum probability", *Pure Mathematics and Applications, Series C*, 2, 223–235.

R. Scozzafava (2000), "The role of probability in statistical physics", *Transport Theory and Statistical Physics*, 29, 107–123.