

Computational Optimization Research Group

Settore Scientifico Disciplinare: Ricerca Operativa, MAT/09

<https://mate.unipv.it/mathopt/>

Stefano Gualandi
Prof. Ass.



Davide Duma
RTDa



Claudio Tomasi
(Postdoc)



Gennaro Auricchio
Postdoc in UK



Computational Optimization Research Group, 2022

Ambrogio



Stochastic Optimization and
Renewable Energy (CESI)

Andrea



Optimization for Computer Vision
in Pharma Industry (SEAVISION)

Eleonora



Computational Optimization with
applications in Genomics (Fondi Eccel.)

Luca



Integer Linear Programming
and Combinatorial
Optimization

Lorenzo



Explainable Machine Learning for
Predictive Maintenance
(FEDEGARI)

Lorenzo



Optimal Transport.
Reconciliation of Probability
Measures (Fondi Eccel.)

Research and Industrial Projects

- Research Project: **Development of a multimodal schedule-based route planner.**
Funded by the European Commission, Joint Research Center, Unit JRC.C.6
Economics of Climate Change, Energy and Transport (ECCET), Seville.
Budget: 135K euro, 2022-2023.
(previous successful small project in 2018, granted 13.5K euro)
- Industrial Project: **Explainable Machine Learning for Predictive Maintenance.**
Funded by Fedegari spa, Italy. Budget: 90K euro, 2021-2024.
- Research Project: **Development of a ready-to-use algorithm for the efficient computation of approximate Kantorovich-Wasserstein distance for large spatial maps.**
Funded by the European Commission, Eurostat, Luxembourg.
Budget: 14.5K euro, 2020-2021.
Outcome: <https://github.com/eurostat/Spatial-KWD/>
- Industrial Project: **Models and Algorithms of Mathematical Optimization for disruptions management in rail freight transportation.**
Funded by Binary srl, Italy. Budget: 45K euro, 2019-2020.
- Industrial Project: **Optimal Resource Allocation in Call Centers.**
Funded by Comdata spa, Italy. Budget: 27.9K euro, 2018-2019.

Lauree Magistrali Plus, LM+, 2018-2022

Aziende che hanno collaborato con il dipartimento

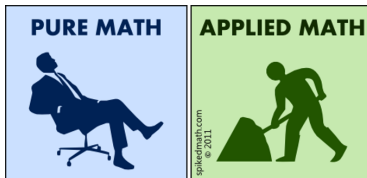


UNIVERSITÀ DI PAVIA



Teaching

- **Operations Research, 9 CFU (6+3).**
2nd semester. Exam: project + oral. Python (*Julia?*)
<http://mate.unipv.it/gualandi/opt4ds/>
- **Numerical Optimization and Data science, 6 CFU (3+3).**
2nd semester. Exam: project + oral. Matlab
(*Altri atenei: Complementi di Ricerca Operativa, o Nonlinear Optimization*)
- **Programmazione 1, 6 CFU.**
1st semester, 1st year semester. Exam: live coding in Python.
- **Integer Programming and Combinatorial Optimization, Ph.D. course.**
Prof. Austin Buchanan, Associate Professor at Oklahoma State University.
May-June, 2022.



Ottimizzazione

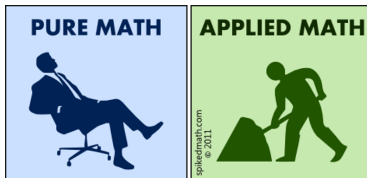
Data una funzione $f : \mathbb{Z}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ e le funzioni $g_i : \mathbb{Z}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ per $i = 1, \dots, k$, si deve trovare il vettore (x^*, y^*) che risolve:

$$z^* = \min f(x, y)$$

$$\text{s.t. } g_i(x, y) \leq 0$$

$$i = 1, \dots, k$$

$$x \in \mathbb{Z}^n, y \in \mathbb{R}^m.$$



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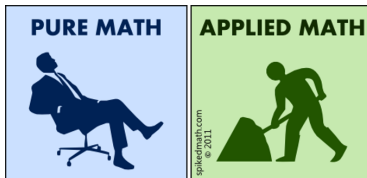
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Se $n = 0, k = 0 \rightarrow$ NONLINEAR (SMOOTH) OPTIMIZATION



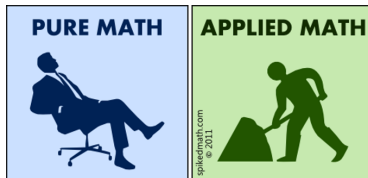
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Se $n = 0, k = 0 \rightarrow$ NONLINEAR (SMOOTH) OPTIMIZATION

Se $n = 0, f(x, y), g_i(x, y)$ convesse $\rightarrow$ CONVEX OPTIMIZATION



Ottimizzazione

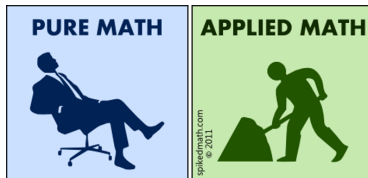
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Se $n = 0, f(x, y), g_i(x, y)$ convesse $\rightarrow$ CONVEX OPTIMIZATION

Se $n \neq 0, f(x, y)$ e $g_i(x, y)$ lineari $\rightarrow$ MIXED INTEGER LINEAR PROGRAMMING



Ottimizzazione

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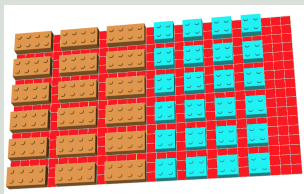
Se $n \neq 0, f(x, y)$ e $g_i(x, y)$ lineari $\rightarrow$ MIXED INTEGER LINEAR PROGRAMMING

Se $m = 0, f(x, y)$ e $g_i(x, y)$ lineari, $x \in \{0, 1\}^n \rightarrow$ COMBINATORIAL OPTIMIZATION

The Lego Production Problem (*inizio corso*)

You are given the following Lego pieces (*raw materials*):

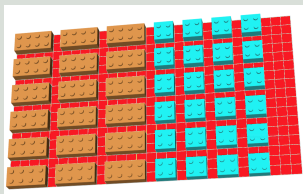
- 18 LARGE pieces (4×2)
- 24 SMALL pieces (2×2)



The Lego Production Problem (*inizio corso*)

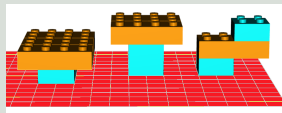
You are given the following Lego pieces (*raw materials*):

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- 24 SMALL pieces (2×2)



You can build and sell two products:

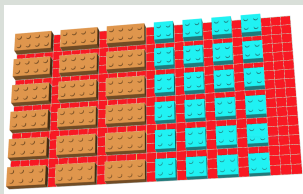
- **Chairs** at 8 kroner, composed of 2 SMALL and 1 LARGE
- **Tables** at 11 kroner, composed of 2 SMALL and 2 LARGE
- **Sofabored** at 15 kroner, composed of 2 SMALL and 3 LARGE pieces



The Lego Production Problem (*inizio corso*)

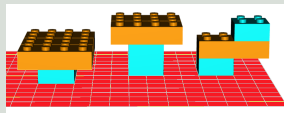
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QUESTION: How many **Chairs**, **Tables**, and **Sofabored** you have to produce to maximize your profit?

Math and Images

Disclaimer: “Images are just numbers!”

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64x64 pixels

MedNIST Challenge

Math and Images

Disclaimer: "Images are just numbers!"



64x64 pixels
MedNIST Challenge



16x16 pixels
Downsampling

Math and Images

Disclaimer: "Images are just numbers!"



64x64 pixels
MedNIST Challenge



16x16 pixels
Downsampling

17	14	13	12	15	14	14	87	82	16	14	14	13	14	16	21
15	12	11	10	104	77	10	123	116	23	101	13	10	12	14	19
13	10	10	11	115	108	13	146	146	63	117	29	8	10	11	17
12	9	90	64	106	140	11	159	145	114	148	21	6	7	10	16
11	7	99	123	85	152	98	184	184	165	161	17	8	8	13	16
11	6	44	135	110	166	152	174	177	168	171	5	8	93	114	15
11	6	5	139	162	153	185	195	189	196	163	5	102	171	139	16
10	5	6	46	161	168	208	218	214	227	177	6	165	173	54	15
9	4	25	22	170	188	212	210	210	230	170	158	207	156	6	11
8	3	241	147	119	217	224	225	225	244	182	215	214	86	6	11
5	1	91	249	228	211	247	253	253	254	234	216	158	5	5	11
5	1	16	186	81	195	255	255	255	255	252	152	5	4	4	11
6	2	1	1	3	234	255	255	255	253	138	4	0	1	3	10
6	2	2	2	5	255	255	255	255	218	27	5	1	2	3	10
6	3	2	2	95	255	251	255	255	115	2	2	1	1	3	10
8	4	4	3	171	255	255	255	255	5	3	3	3	3	4	10

Image as matrix of
numbers

Math, Color Images, and Tensors



RGB Image

Math, Color Images, and Tensors



RGB Image



Red channel



Green channel



Blu channel

Images and Point Clouds

RGB Image

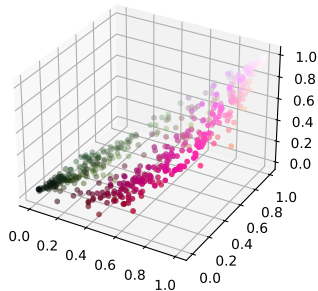


Images and Point Clouds

RGB Image



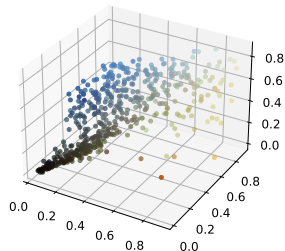
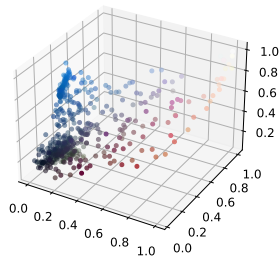
Sampling of pixels by colors in 3D



How to generate a fake van Gogh for Borgo Ticino

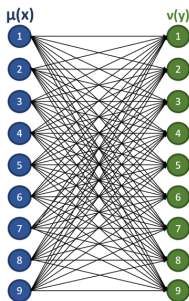


How to generate a fake van Gogh for Borgo Ticino



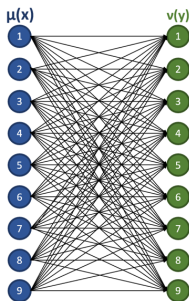
Computing the KWD between two spatial distributions

Given two empirical distributions μ and ν defined over a space X , we can define a complete bipartite graph:



Computing the KWD between two spatial distributions

Given two empirical distributions μ and ν defined over a space X , we can define a complete bipartite graph:



Using the bipartite graph $B = (V \cup W, E)$, we can write the LP *bipartite* model:

$$\min \sum_{ij \in E} c_{ij} \pi_{ij} \quad (1)$$

$$\text{s.t.} \quad \sum_{ij \in E} \pi_{ij} = \mu_i, \quad \forall i \in V, \quad (2)$$

$$\sum_{ij \in E} \pi_{ij} = \nu_j, \quad \forall j \in W, \quad (3)$$

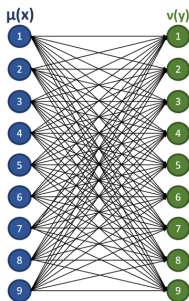
$$\pi_{ij} \geq 0, \quad \forall \{i, j\} \in E. \quad (4)$$

where the cost are defined as:

$$c(i, j) := d(x_i, x_j) = \|x_i - x_j\|_2. \quad (5)$$

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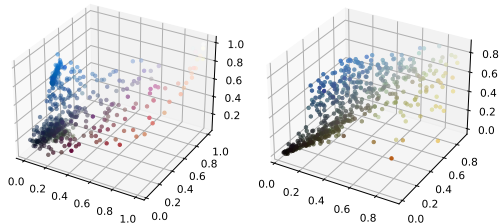
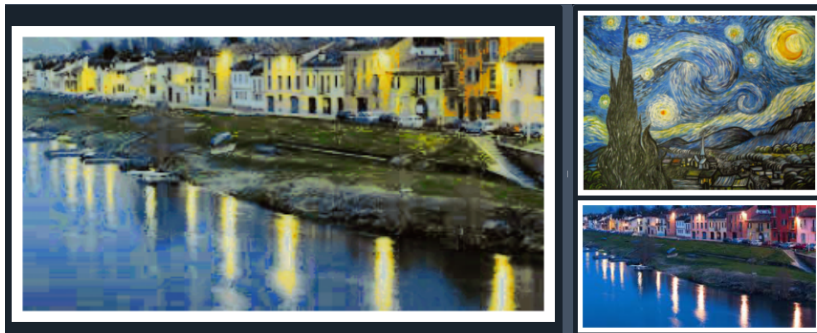
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REMARK

We need to solve an optimization problem to compute the Kantorovich Wasserstein Distance. We use an optimal transportatino plan to mapping the colors of van Gogh.

How to generate a fake van Gogh for Borgo Ticino



Old Math, New (Snapchat?) Applications



Old Math, New (Snapchat?) Applications



M É M O I R E
S U R L A
T H É O R I E D E S D É B L A I S
E T D E S R E M B L A I S.

Par M. M O N G E.

Lorsqu'on doit transporter des terres d'un lieu dans un autre, on a coutume de donner le nom de *Déblai* au volume des terres que l'on doit transporter, & le nom de *Remblai* à l'espace qu'elles doivent occuper après le transport.

Le prix du transport d'une molécule étant, toutes choses d'ailleurs égales, proportionnel à son poids & à l'espace qu'on lui fait parcourir, & par conséquent le prix du transport total devant être proportionnel à la somme des produits des molécules multipliées chacune par l'espace parcouru, il s'ensuit que le déblai & le remblai étant donnés de figure & de position, il n'est pas indifférent que telle molécule du déblai soit transportée dans tel ou tel autre endroit du remblai, mais qu'il y a une certaine distribution à faire des molécules du premier dans le second, d'après laquelle la somme de ces produits sera la moindre possible, & le prix du transport total

Old Math, New (Snapchat?) Applications

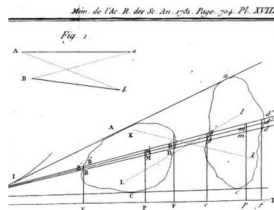


M É M O I R E S U R L A T H É O R I E D E S D É B L A I S E T D E S R E M B L A I S.

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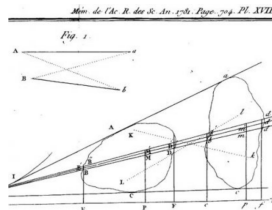
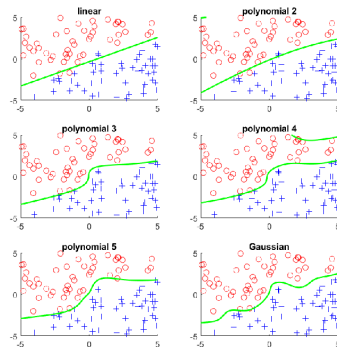
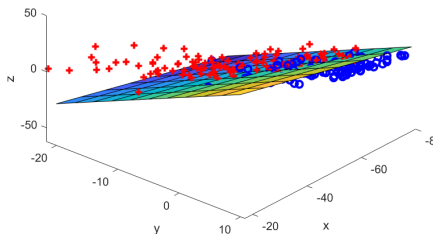


Figure 1. L. V. Kantorovich in his youth.

Esempio: Ovarian Cancer Detection with Machine Learning

Predicting the presence of ovarian cancer from 4.000 markers
(dimensionality reduction + classification)



Possibili argomenti di tesi

Panoramica su tesi concluse di recente:
<http://mate.unipv.it/gualandi/for-students>

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Possibili argomenti:

- Algoritmo di Trasporto Ottimo per problemi di Natural Language Processing
- Problemi di Ottimizzazione per l'architettura ottima di reti neurali di Deep Learning
- Algoritmi di trasporto multimodale
- Algoritmi di Graph coloring e di Total Matching per Grafi Sparsi

Possibili argomenti di tesi

Possibili argomenti:

- Modelli di Simulazione a Eventi Discreti per la valutazione di policy nella gestione dei servizi sanitari (AnyLogic, MATLAB & Simulink, Salabim)
- Tecniche di Process Mining per l'analisi dei processi aziendali ospedalieri o farmaceutici (ProM, C)
- Algoritmi di Ottimizzazione Online

Questions?

Thanks for your attention.