

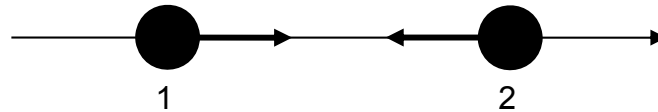


Collisions and shape memory alloys

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When predicting the motion of mechanical systems,
the mechanical quantities, velocities, temperatures,...
can evolve very rapidly and one may assume that they are
discontinuous with respect to time and space.

Discontinuities with respect to time



2 balls with velocities U_1 and U_2 collide
velocities U_1^- and U_2^- before collision
velocities U_1^+ and U_2^+ after collision

The system made of two points is deformable

We choose the velocity of deformation

$$D = U_2 - U_1$$

Work of the interior forces

$$-P^{int} \frac{D^+ + D^-}{2}$$

The virtual work principle gives the equations of motion

$$m_2(U_2^+ - U_2^-) = -P^{int}, \quad m_1(U_1^+ - U_1^-) = P^{int}$$

A constitutive law is needed. It satisfies

$$P^{int} \frac{D^+ + D^-}{2} \geq 0$$

It describes the physical properties of the balls

An example

$$\Phi = k \left(\frac{D^+ + D^-}{2} \right)^2 + I_+(D^+)$$

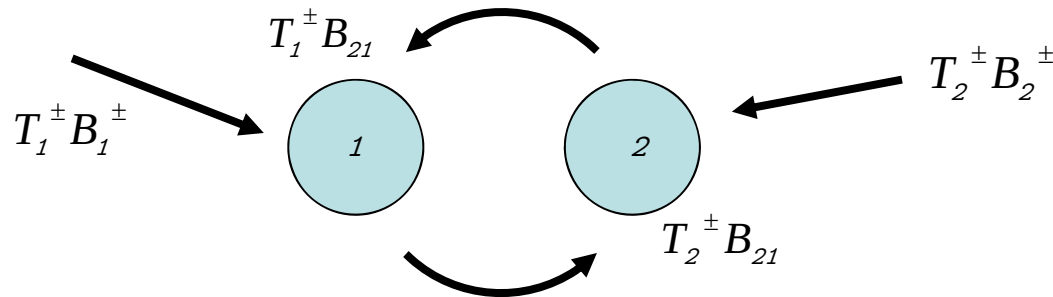
The indicator function I_- takes into account the impenetrability condition

$$D^+ = U_2^+ - U_1^- \geq 0,$$

$$P^{int} \in \partial \Phi \left(\frac{D^+ + D^-}{2} \right) = k (D^+ + D^-) + \partial I_-(D^+)$$

Temperatures T_2^+ , T_2^- and T_1^+ , T_1^-

$$\delta T = \frac{T_2^+ + T_2^-}{2} - \frac{T_1^+ + T_1^-}{2}$$



Constitutive laws satisfy

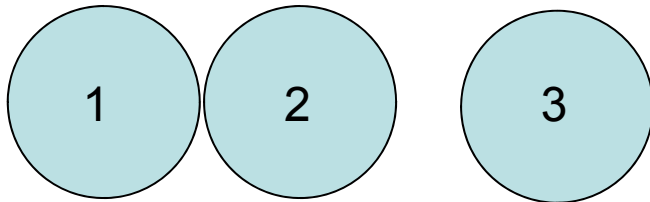
$$P^{int} \frac{D^+ + D^-}{2} - \delta T \frac{B_{21} - B_{12}}{2} \geq 0$$

In any example, the constitutive law for P^{int} and $B_{21} - B_{12}$ are simple:
 linear functions of $D^+ + D^-$ and δT
 plus the reactions to the impenetrability conditions

Examples without temperature:

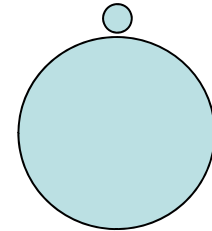
3 balls or coins: the Newton's cradle; the superball phenomenon.

4 balls (Ben Mekki, Tor Vegata-ENPC).



Tennis table ball: 1

Soccer ball: 2

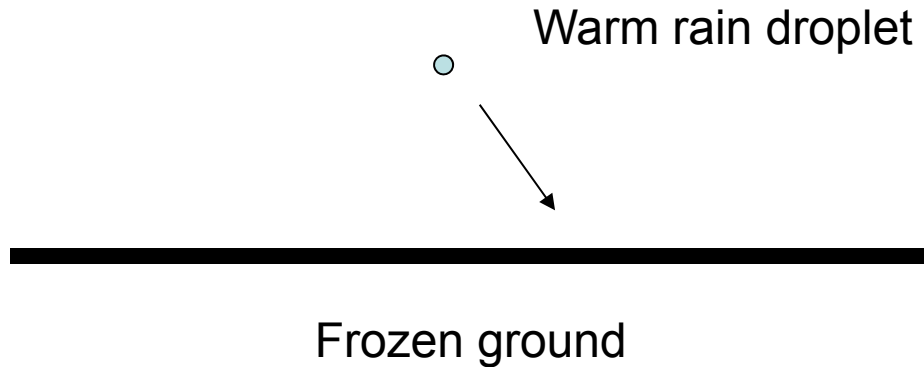


Floor: 3

One of the deformation velocities is $U_3 - U_1$
introducing at a distance interactions

Example with the temperature

Warm rain falling on a frozen ground.
Does the rain freeze and give black ice?

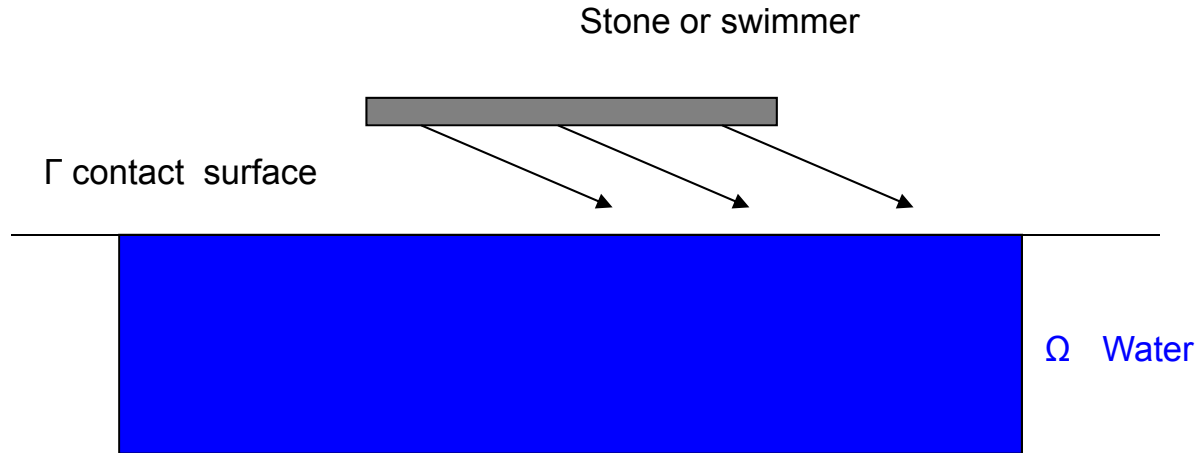


Closed form solutions: **dissipated work**,
heat capacity, thermal conductivity intervene

$$\theta_2^- \leq -\frac{\mathcal{T} + 2L}{\lambda T_0} - \frac{2C}{\lambda T_0} \theta_1^- ,$$

$\mathcal{T} > 0$ is the collision dissipated work,
 $\theta_1^- > 0$ is the warm rain temperature before collision
(A. M. Caucci, University of Bologna)

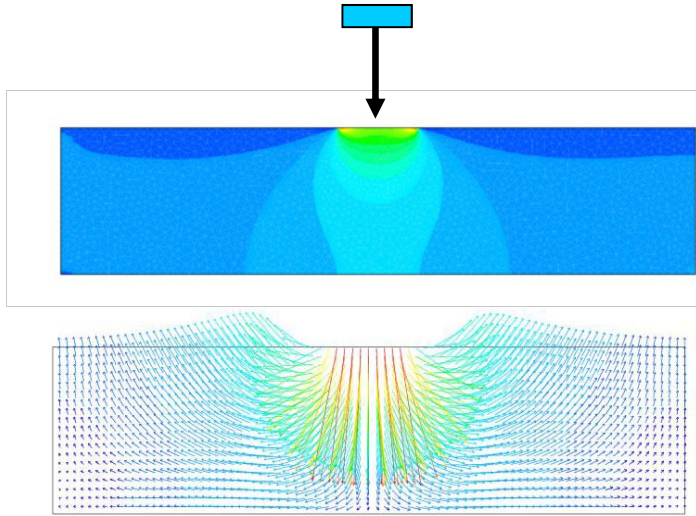
Collisions of deformable bodies, E. Dimnet LCPC,
 Collisions of solids and fluids,
 E. Dimnet, R. Gormaz, J. San Martin University of Chile



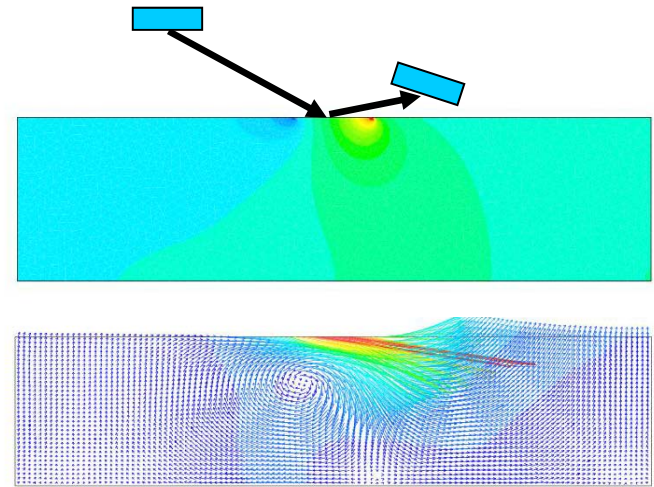
Work of the interior forces

$$\begin{aligned}
 & - \int_{\Omega} \Sigma : E \left(\frac{\vec{U}^+ + \vec{U}^-}{2} \right) d\Omega \\
 & - \int_{\Gamma} \vec{R} \cdot \frac{(\vec{U}_{water}^+ - \vec{U}_{stone}^+) + (\vec{U}_{water}^- - \vec{U}_{stone}^-)}{2} d\Gamma
 \end{aligned}$$

Linear constitutive laws plus impenetrability condition



Belly flop

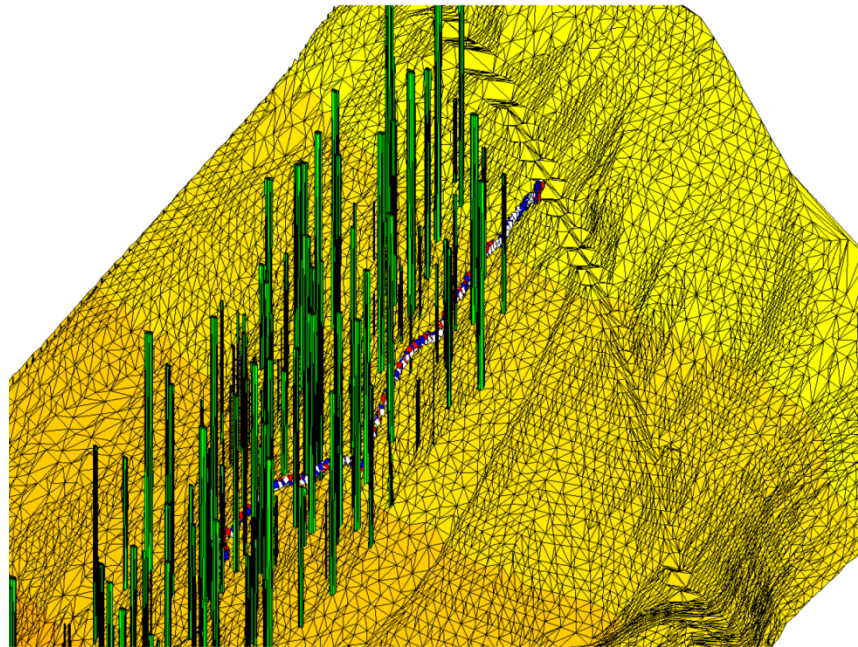


Skipping stone

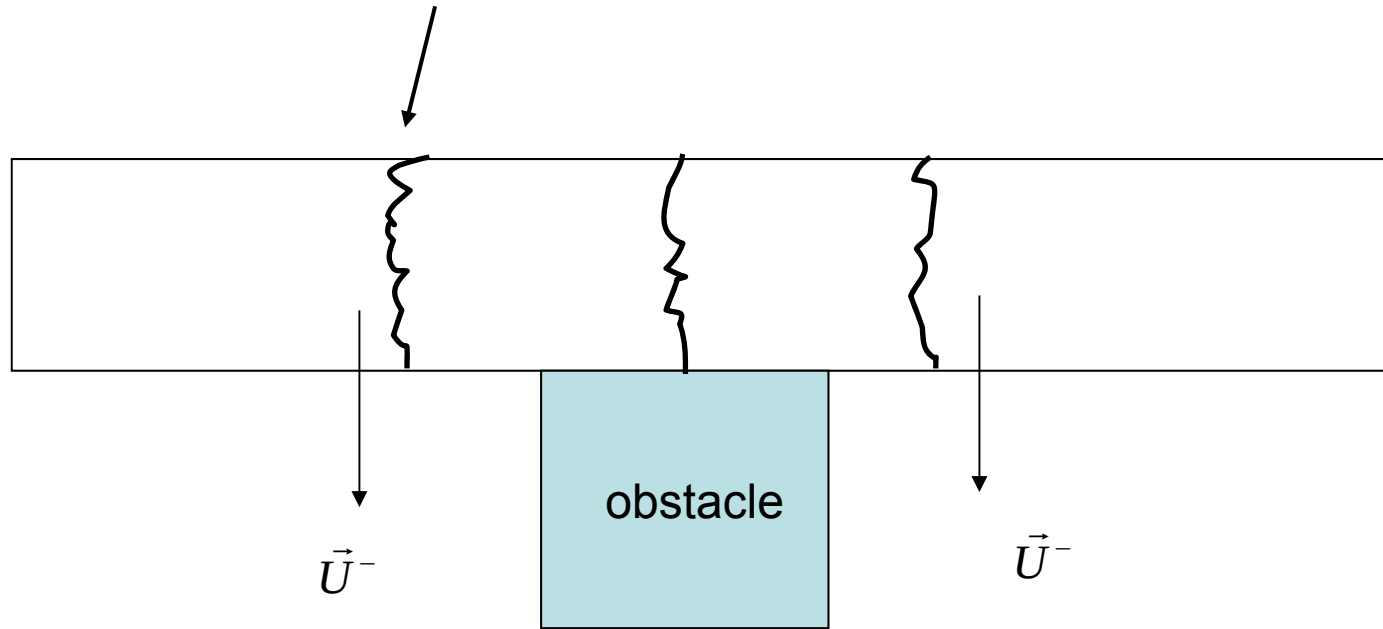
The constitutive laws are the same.
The intensity of the horizontal velocity is the only difference

A collision is always dissipative.
The impenetrability reaction works. It is not workless.

The impenetrability can be conditional:
a tree can be broken by an avalanching rock (E. Dimnet LCPC) .



Positions Γ of the fractures are unknown



Collision is instantaneous.

There are velocities before collision $\vec{U}^-(x)$ and velocities $\vec{U}^+(x)$ after collision

Fractures result from the collision. Thus velocity $\vec{U}^+(x)$ is
a discontinuous function of x

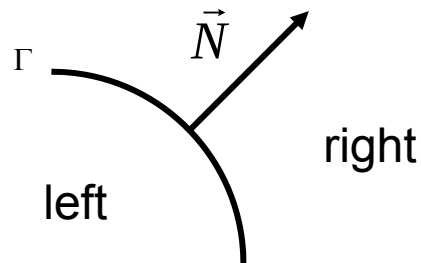
The velocities are discontinuous:

with respect to time

$$\vec{U}^+(x) - \vec{U}^-(x)$$

with respect to space

$$[\vec{U}^+(x)] = \vec{U}_r^+(x) - \vec{U}_l^+(x) = [\vec{U}^+(x) + \vec{U}^-(x)]$$



We choose the velocities of deformation. They are the important quantities of the theory,

$$E(\frac{\vec{U}^+ + \vec{U}^-}{2}), \text{ in } \Omega \setminus \Gamma,$$

the usual strain rate where there is no fracture, and

$$\left[\vec{U}^+ + \vec{U}^- \right],$$

on fracture Γ and on the contact surface $\partial\Omega_1$ of the solid with the obstacle, which is also a fracture. It results the work of the interior forces

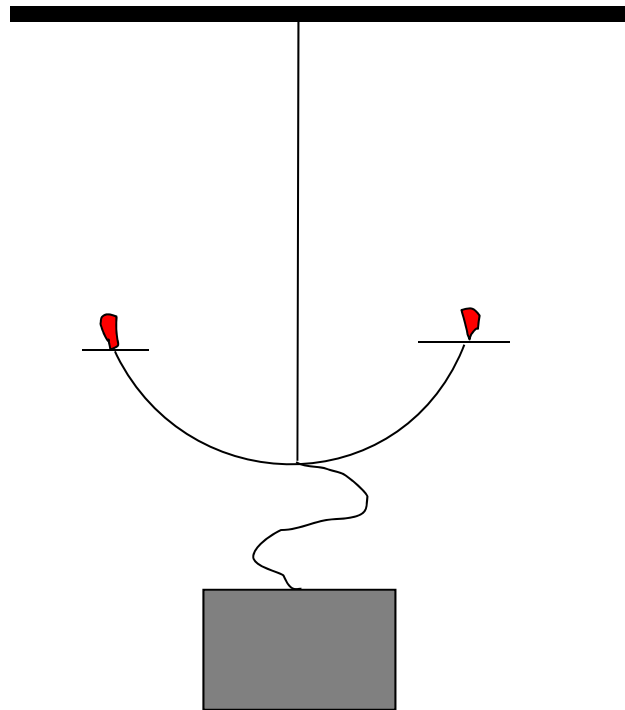
$$\begin{aligned} T_{int} = & - \int_{\Omega \setminus \Gamma} \Sigma : E(\frac{\vec{V}^+ + \vec{V}^-}{2}) d\Omega - \int_{\partial\Omega_1} \vec{R} \cdot (\frac{\vec{V}^+ + \vec{V}^-}{2}) d\Gamma \\ & + \int_{\Gamma} \vec{R} \cdot \left[\frac{\vec{V}^+ + \vec{V}^-}{2} \right] d\Gamma. \end{aligned}$$

The percussion stress Σ and the percussions $\vec{R}$ are the interior forces.

The equations of motion are

$$\begin{aligned}\operatorname{div} \Sigma &= \rho(\vec{U}^+ - \vec{U}^-), \text{ in } \Omega \setminus (\Gamma \cup \partial\Omega_1), \\ [\Sigma] \vec{N} &= 0, \quad \Sigma \vec{N} = \vec{R}, \text{ on } \Gamma, \\ \Sigma \vec{N} &= -\vec{R}, \text{ on } \partial\Omega_1, \quad \Sigma \vec{N} = 0, \text{ on } \partial\Omega_2.\end{aligned}$$

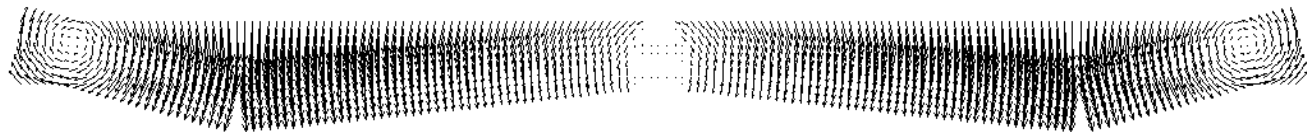
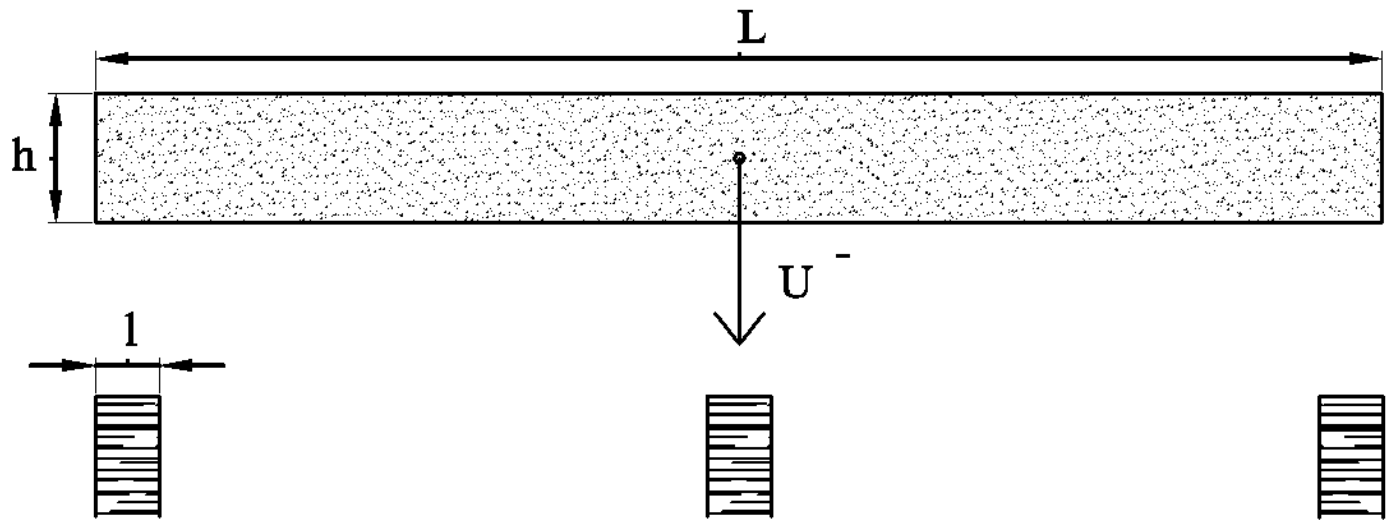
The constitutive laws take into account that fracturation is
not a progressive phenomenon



There are closed form solutions for 1-D problems:

A stone is tied to a chandelier.

Work with Elena Bonetti, Pavia University



Work with Francesco Freddi, Parma University

Collisions and shape memory alloys

State quantities

ε small deformations

$\vec{\beta} = (\beta_1, \beta_2, \beta_3)$ phase volume fractions

β_1, β_2 martensites, β_3 austenite

T temperature

Velocities

$\vec{U}$ macroscopic velocity

$\dot{\vec{\beta}} = \frac{d\vec{\beta}}{dt}$ volume fractions velocities

At collision time $\vec{U}, \vec{\beta}, T$ are discontinuous

$$A^+(t) = \lim_{\Delta t \rightarrow 0, \Delta t > 0} A(t + \Delta t)$$

$$A^-(t) = \lim_{\Delta t \rightarrow 0, \Delta t > 0} A(t - \Delta t)$$

$$[A] = A^+ - A^-, \quad \bar{A} = \frac{A^+ + A^-}{2}$$

Predictive theory to get $\vec{U}^+, \vec{\beta}^+, T^+$

depending on $\vec{U}^-, \vec{\beta}^-, T^-$

Free energy

$$\begin{aligned}\Psi(\varepsilon = 0, \vec{\beta}, \text{grad } \vec{\beta}, T) \\&= I_C(\vec{\beta}) + \frac{k}{2}(\text{grad } \vec{\beta})^2 \\&\quad - CT \log T - \beta_3 \frac{L}{T_0}(T - T_0)\end{aligned}$$

$$C = \{(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3; 0 \leq \gamma_i \leq 1; \gamma_1 + \gamma_2 + \gamma_3 \leq 1\}$$

Principle of virtual work

Virtual velocities $\vec{V}^+, \vec{V}^-, [\vec{\delta}]$

$$\forall \vec{V}^+, \vec{V}^-, [\vec{\delta}], \quad W_{acc} = W_{int} + W_{ext}$$

$$W_{acc} = \int_{\Omega} \rho [\vec{U}] \cdot \frac{\vec{V}^+ + \vec{V}^-}{2} d\Omega$$

$$W_{int} = - \int_{\Omega} \Sigma : D\left(\frac{\vec{V}^+ + \vec{V}^-}{2}\right) d\Omega - \int_{\Omega} \vec{B} \cdot [\vec{\delta}] + H : \text{grad} [\vec{\delta}] d\Omega$$

$$W_{ext} = - \int_{\partial\Omega} \vec{P} \cdot \frac{\vec{V}^+ + \vec{V}^-}{2} d\Gamma$$

Equations of motion

$$\rho \left[\vec{U} \right] = \operatorname{div} \Sigma, \text{ in } \Omega, \quad \Sigma \vec{N} + \vec{R} = 0, \text{ on } \partial\Omega$$

$$0 = -\vec{B} + \operatorname{div} H, \text{ in } \Omega, \quad H \vec{N} = 0, \text{ on } \partial\Omega$$

Energy balance

$$[e] + \operatorname{div}(T^+ \vec{Q}^+ + T^- \vec{Q}^-)$$

$$= \Sigma : D\left(\frac{\vec{U}^+ + \vec{U}^-}{2}\right) + \vec{B} \cdot [\vec{\beta}] + H : \operatorname{grad} [\vec{\beta}],$$

$$(T^+ \vec{Q}^+ + T^- \vec{Q}^-) \cdot \vec{N} = 0, \text{ on } \partial\Omega$$

Second law

$$[s] + \operatorname{div}(\vec{Q}^+ + \vec{Q}^-) \geq 0,$$

Pseudo-potential of dissipation

$$\begin{aligned} & \Phi(D(\frac{\vec{U}^+ + \vec{U}^-}{2}), [\vec{\beta}], \text{grad} [\vec{\beta}], \text{grad } \bar{T}, \bar{T}) \\ &= k_v \left(D(\frac{\vec{U}^+ + \vec{U}^-}{2}) \right)^2 + \frac{c}{2} [\vec{\beta}]^2 \\ &+ \frac{\nu}{2} \text{grad} [\vec{\beta}]^2 + \frac{\lambda}{2\bar{T}} (\text{grad } \bar{T})^2 + I_0([\beta_1 + \beta_2 + \beta_3]) \end{aligned}$$

Constitutive laws

$$\Sigma = k_v D(\vec{U}^+ + \vec{U}^-),$$

$$\vec{B} = \begin{vmatrix} -CT^+ \log T^+ \\ -CT^+ \log T^+ \\ -\frac{l_a}{T_0}(T^+ - T_0) - CT^+ \log T^+ \end{vmatrix} + \partial I_C(\vec{\beta}^+) + c[\beta_i] - P \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$-P \in \partial I_0([\beta_1 + \beta_2 + \beta_3]) = \mathbb{R},$$

$$H = k \operatorname{grad} \vec{\beta}^+ + v \operatorname{grad} [\beta_i],$$

$$-\vec{Q} = \vec{Q}^+ = \vec{Q}^- = \frac{\lambda}{2\bar{T}} \operatorname{grad} \bar{T}.$$

Equations for $\vec{U}^+$, $\vec{\beta}^+$ and T^+

$$-P \in \partial I_0([\beta_1 + \beta_2 + \beta_3]) = \mathbb{R},$$

$$\rho \vec{U}^+ - k_v \Delta \vec{U}^+ = \rho \vec{U}^- + k_v \Delta \vec{U}^-, \text{ in } \Omega,$$

$$c \left[\vec{\beta} \right] - v \Delta \left[\vec{\beta} \right] - k \Delta \vec{\beta}^+ + \partial I_C(\vec{\beta}^+) + \left| \begin{array}{c} -P - CT^+ \log T^+ \\ -P - CT^+ \log T^+ \\ -\frac{l_a}{T_0}(T^+ - T_0) - P - CT^+ \log T^+ \end{array} \right| = 0$$

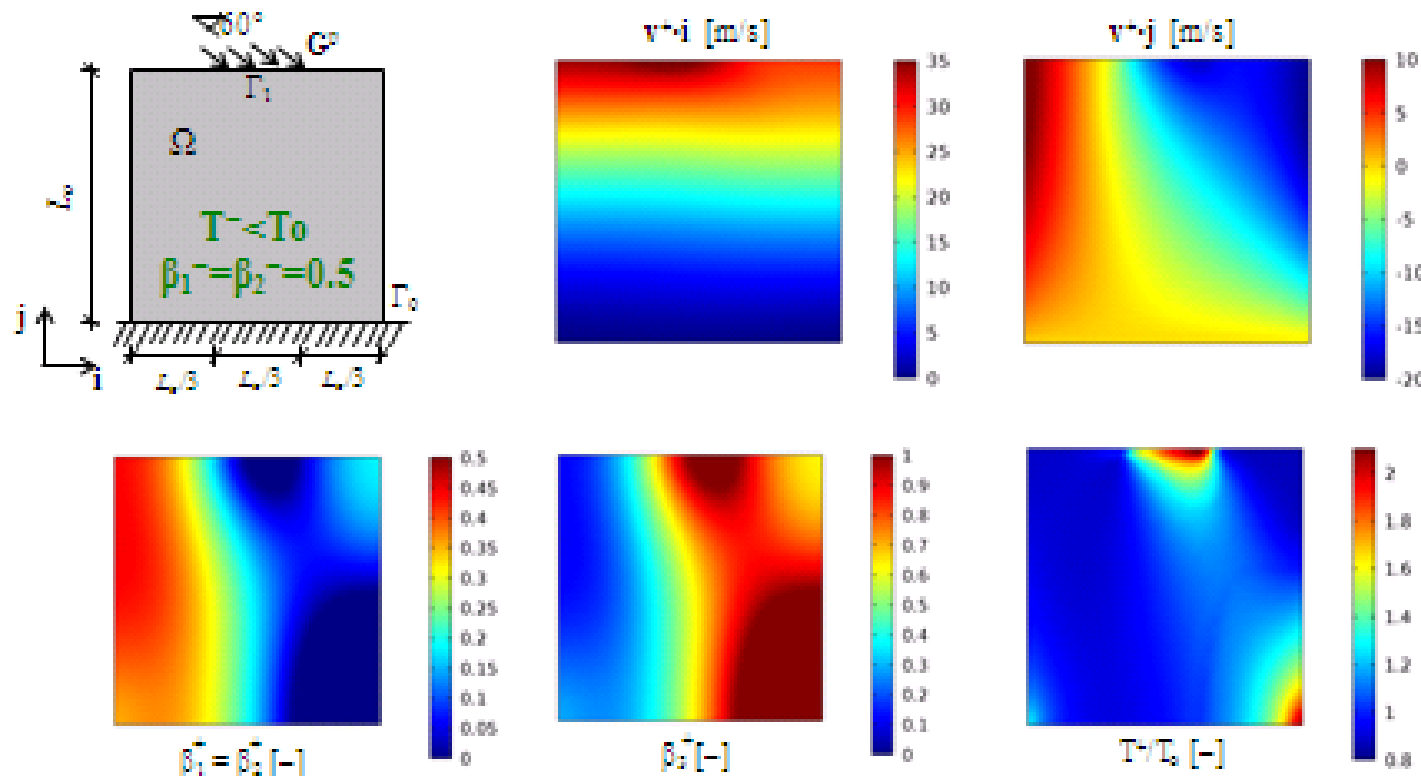
$$\begin{aligned} & (\beta_1^- + \beta_2^- + \beta_3^-)C[T] + l_a[\beta_3] - \lambda \Delta T \\ & = \Sigma : D\left(\frac{\vec{U}^+ + \vec{U}^-}{2}\right) + B_i^p[\beta_i] + \vec{H}_i^p \cdot \text{grad}[\beta_i], \text{ in } \Omega, \end{aligned}$$

plus boundary conditions

Applications

The results from the collision SMA predictive theory are the input data for the thermomechanical smooth problem describing SMA evolution (Frémond model).

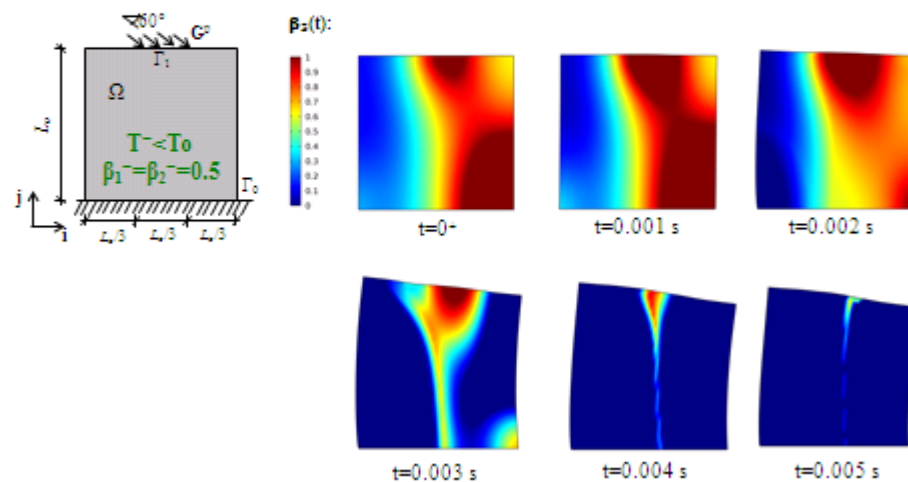
- Impact and post-collision behavior of a 2D block:



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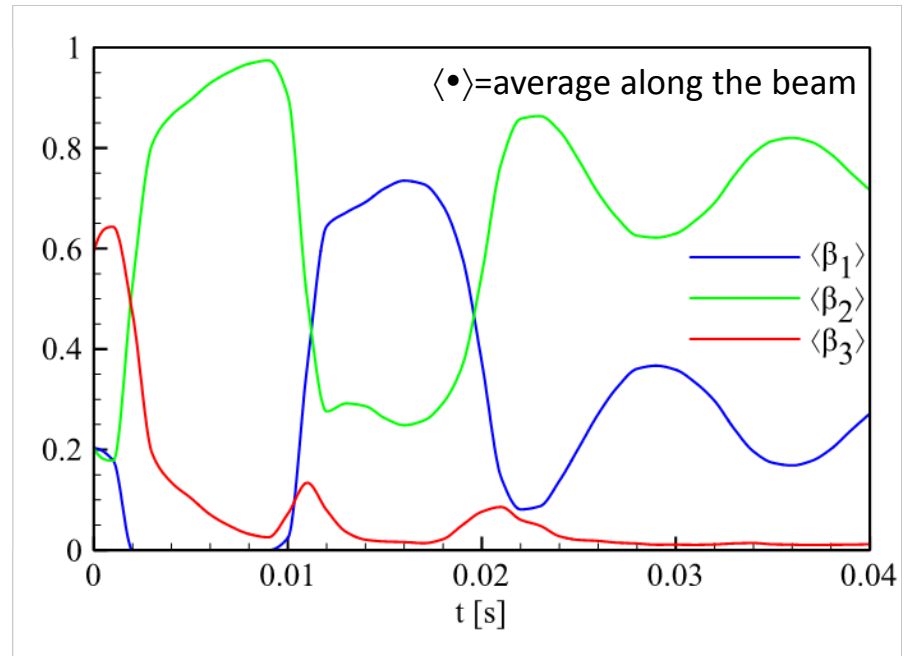
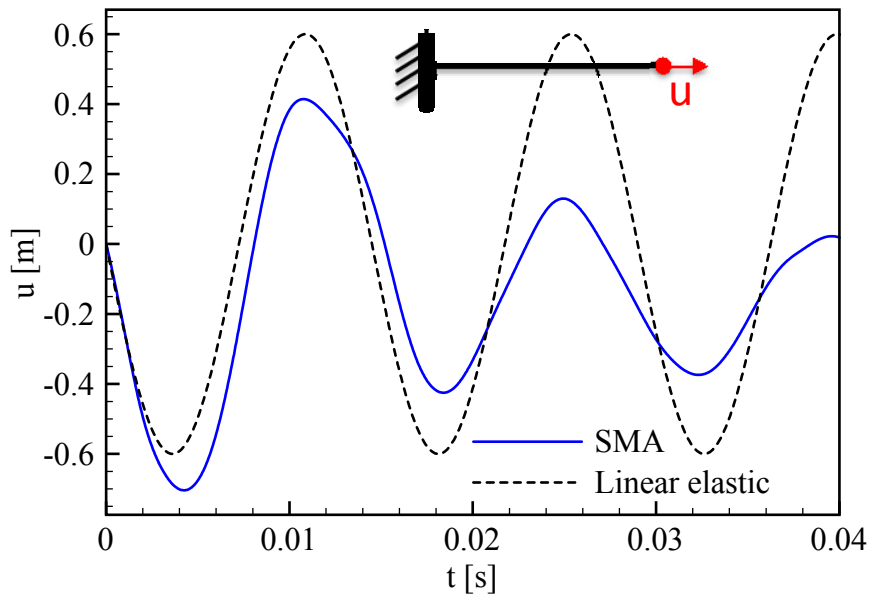
Applications

The results from the collision SMA predictive theory are the input data for the thermomechanical smooth problem describing SMA evolution (Frémond model).

- Impact and post-collision behavior of a 1D beam structure:



$$T^- < T_o, \beta_1^- = \beta_2^- = 0.5$$



Computational step size is limited only
by the smooth evolution problem (structural natural frequency).

SMA constitutive modeling: collisions

Impact behavior depends on classical SMAs parameters and on:

c_c : collision viscosity of phase change;

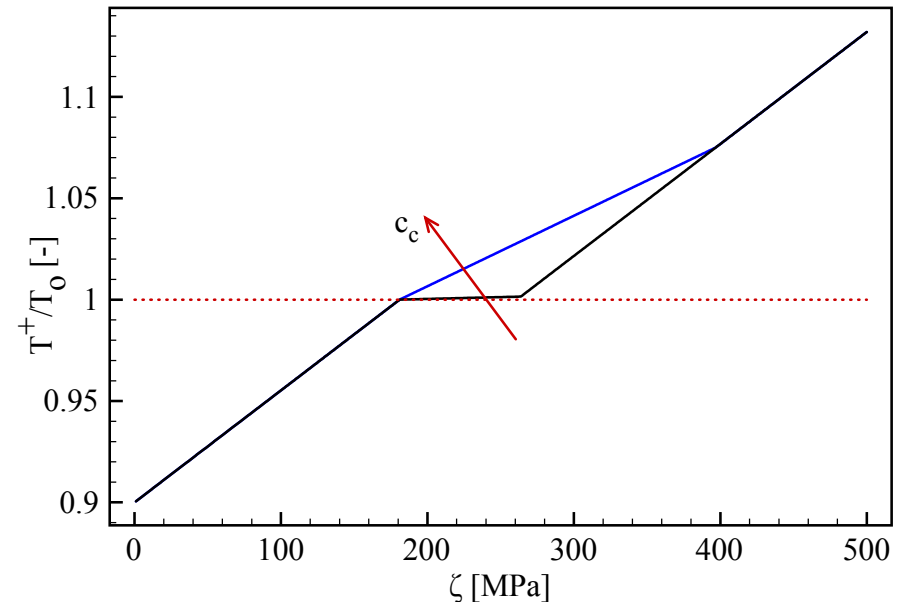
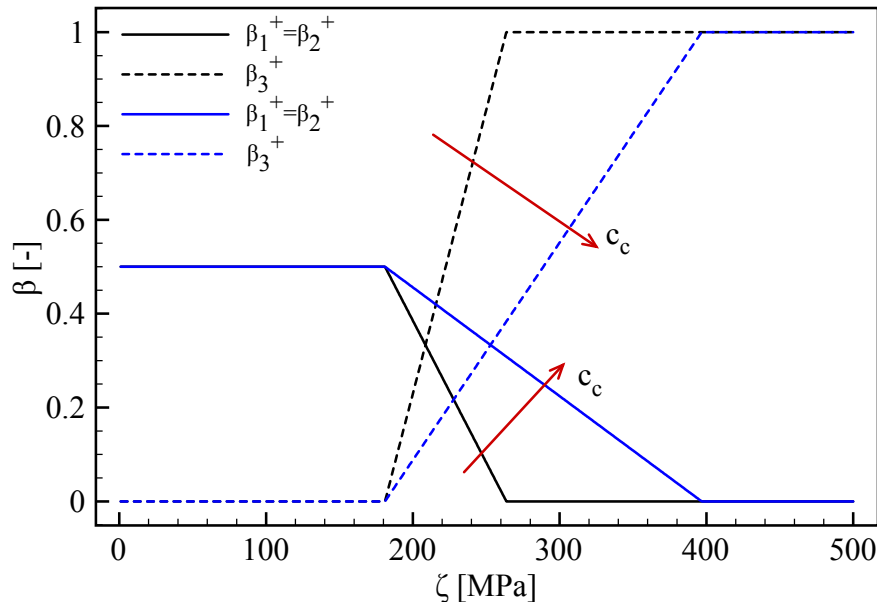
v_c : collision viscosity of phase change diffusion;

k_v : collision viscosity of velocities.

Lower c_c ,
easier is the phase change

Simplified assumptions:

- $T^- < T_o$, $\beta_1^- = \beta_2^- = 0.5$
- Homogenous case: $\nabla \underline{T} = 0$, $\nabla \underline{\beta} = \mathbf{0}$
- Dissipated power $\zeta = \underline{\sigma}^p : \underline{\epsilon}(\underline{v})$ is known



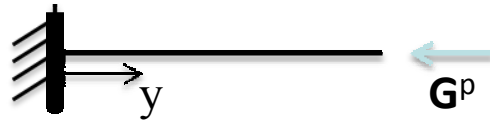
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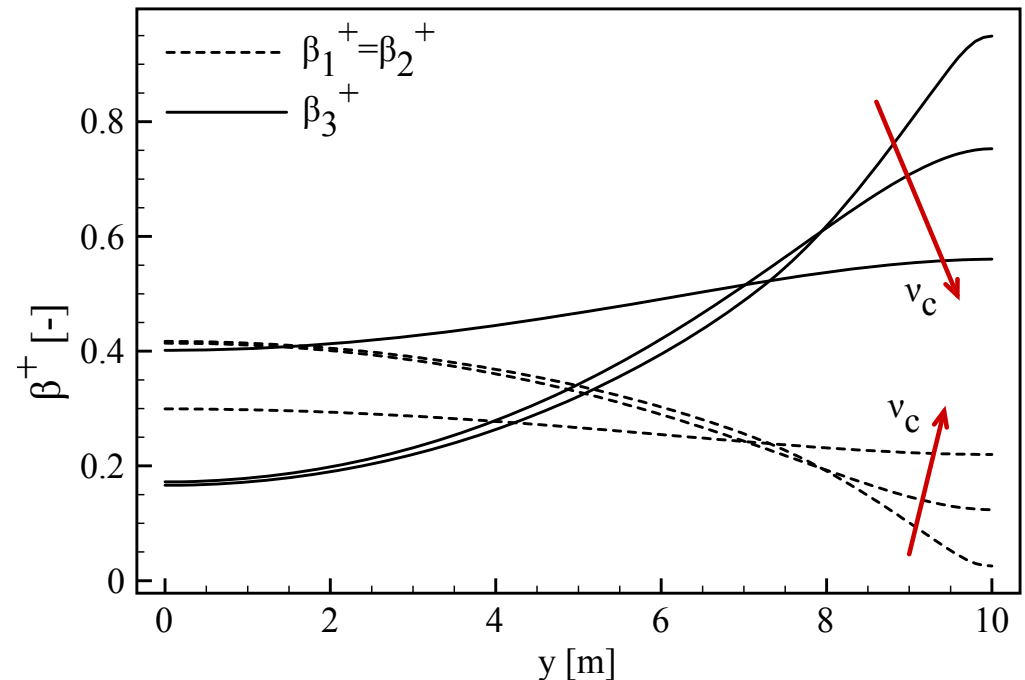
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Higher v_c
more uniform is the
phase change



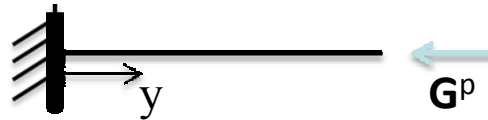
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$$T^- < T_o, \beta_1^- = \beta_2^- = 0.5$$

Higher $k_v \Rightarrow$ Smaller the post-collision velocity $\Rightarrow$ Lower is the phase change

