

New modeling, analysis, and computations for thermal retraction

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Joint work with

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Diffuse Interface Models - DIMO2013
Levico, 12/9/2013

We are interested in ...

Shape memory
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We are interested in ...

Let $Q = \Omega(\subset \mathbf{R}^n) \times (0, T)$ ($n \leq 3$)

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f$$

$$\partial \phi(\chi_t) - \Delta \chi + \partial \beta(\chi)$$

$$\ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

$$u = u_\Gamma, \partial_n \chi = 0 \quad \text{in } \partial\Omega$$

$$\chi(0) = \chi_0$$

with

- ▶ k smooth, $0 < k_* \leq k \leq k^*$, $k(1) > k(0)$
- ▶ G smooth, with $G(1) = 0$ and $G' < 0$
- ▶ ϕ, β proper, convex, l.s.c.
- ▶ suitable growth conditions for ϕ
- ▶ $\operatorname{dom} \beta \subseteq [a, b]$

We are interested in ...

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f$$

$$\partial\phi(\chi_t) - \Delta\chi + \partial\beta(\chi)$$

$$\ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

- ▶ $k(\chi, \theta) = \left(\frac{\chi}{k_1} + \frac{(1-\chi)}{k_2} \right)^{-1} \exp(2E/(\theta - \theta_0)),$
 $k_1 > k_2 > 0, E > 0, \theta > \theta_0$ (temperature)
- ▶ $G(\chi, \theta) = C \exp(E/(\theta - \theta_0))(1 - \chi)$
- ▶ the pseudo-potential of dissipation [Moreau]

$$\phi(\chi_t) = \frac{1}{2}|\chi_t|^2 + H(\chi_t)$$

- ▶ $H(\chi_t) = 0$ if $\chi_t \geq 0$
- ▶ $H(\chi_t) = -\eta(\theta)\chi_t$ if $\chi_t < 0$ ($\eta \geq 0, \eta(0) = 0$)

We are interested in ...

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f$$

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$$\ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

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 $k_1 > k_2 > 0, E > 0, \theta > \theta_0$ (temperature)
- ▶ $G(\chi) = C \exp(E/(\theta - \theta_0))(1 - \chi)$
- ▶ $\phi(\chi_t) = \frac{1}{2}|\chi_t|^2 + H(\chi_t)$ and ∂H is



We are interested in ...

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f$$

$$\partial\phi(\chi_t) - \Delta\chi + \partial\beta(\chi)$$

$$\ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

- ▶ $\phi(\chi_t) = \frac{1}{2}|\chi_t|^2 + H(\chi_t)$
- ▶ $\beta(\chi) = I_{[0,1]}(\chi)$, with
 - ▶ $I_{[0,1]}(\chi) = 0$ if $\chi \in [0, 1]$
 - ▶ $I_{[0,1]}(\chi) = +\infty$ otherwise

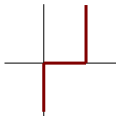
We are interested in ...

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- ▶ $\phi(\chi_t) = \frac{1}{2}|\chi_t|^2 + H(\chi_t)$
- ▶ $\beta(\chi) = I_{[0,1]}(\chi)$ and $\partial\beta$ is



Shape memory effects in polymers

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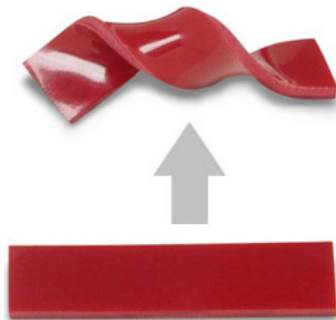
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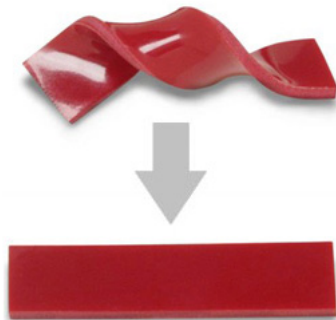
The material is deformed at high temperature



Once it is cooled the deformation is permanent

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Then, for high temperature it recovers the original shape



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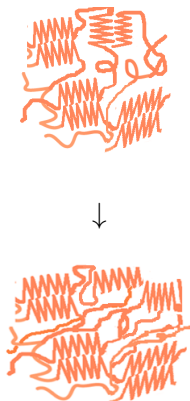
- good and promising alternative to SMA (less expensive, cheaper process for production, possible biocompatible and biodegradable + good mechanical properties). E.g., a very common application: packaging



- however lack of modeling and analytical investigation

Shape memory effects in polymers

Polymeric materials can show **thermal retraction on heating** when the molecular structure has been **oriented by mechanical loads**



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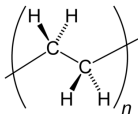
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Polyethylene LDPE

→ density depends on the possibility of creating crystalline configuration

- Polymers: a large molecule (macromolecule) composed of repeating structural units, connected by covalent chemical bonds



- presence of two different configurations **amorphous + crystalline**
- **different physical properties** and different transitions temperature

$$\theta_0 < \theta < \theta_m$$

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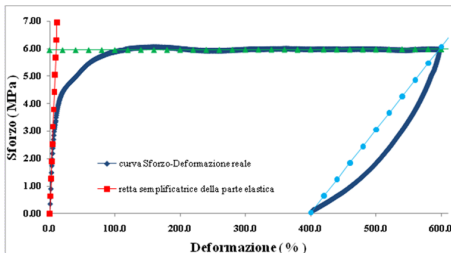
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Thermal retraction

- ▶ the material is deformed at a given temperature θ_d and then unloaded
- ▶ for $\theta < \theta_d$ the deformation (at zero stress) is fixed



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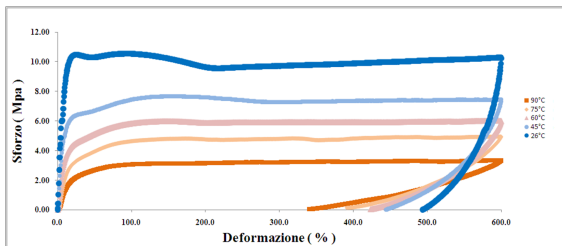
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Thermal retraction

- ▶ the material is deformed at a given temperature θ_d and then unloaded
- ▶ for $\theta < \theta_d$ the deformation (at zero stress) is fixed
- ▶ when the temperature $\theta \geq \theta_d$ a retraction occurs



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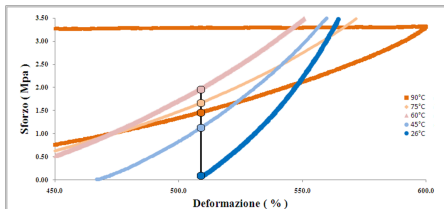
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Thermal retraction

- ▶ the material is deformed at a given temperature θ_d and then unloaded
- ▶ for $\theta < \theta_d$ the deformation (at zero stress) is fixed
- ▶ when the temperature $\theta \geq \theta_d$ a retraction occurs (or a residual stress)



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A phase transition approach

As we are interested in a **macroscopic description**
(characterizing measurable mechanical properties of the material)

- ▶ model capturing **micro-processes towards macroscopic effects**:

A phase transition approach

As we are interested in a **macroscopic description** (characterizing measurable mechanical properties of the material)

- ▶ model capturing **micro-processes** towards **macroscopic effects**: **phase transitions models**

Assuming

- ▶ the proportions of the crystalline and amorphous phases are fixed, at a **given temperature** $\theta_0 < \theta < \theta_m$
- ▶ a phase parameter χ is introduced to characterize the **non-orientation of the amorphous phase**

$$\chi \in [0, 1]$$

where $\chi = 0$ completely oriented, $\chi = 1$ **non-oriented**.

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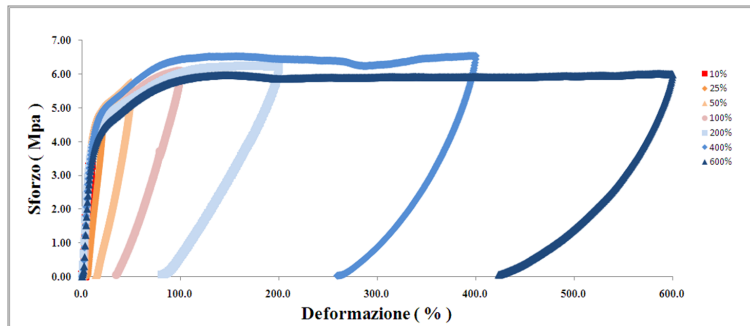
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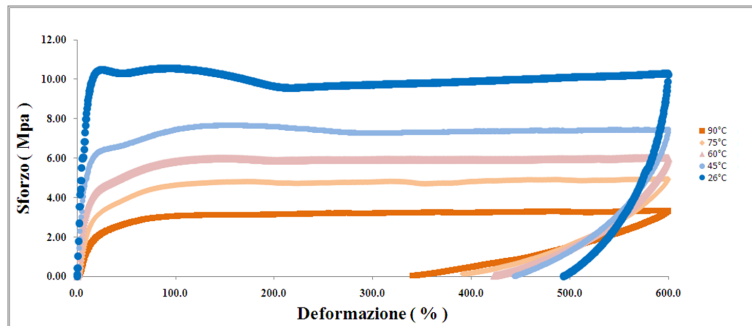
References

- ▶ for small deformation $\chi = 1$ the material is elastic
- ▶ at yield value a phase transition occurs $\chi \searrow 0$
- ▶ unloading for $\chi < 1$ a residual strain occurs if the stress $\sigma = 0$

The stress-strain relation in terms of χ

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The equations

The resulting PDE system is recovered by a variational principle: the generalized principle of virtual powers [Frémond].

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f$$

$$\chi_t + \partial H(\chi_t) - \Delta \chi + \partial I_{[0,1]}(\chi)$$

$$\ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

$$u = u_\Gamma, \partial_n \chi = 0 \quad \text{in } \partial\Omega$$

$$\chi(0) = \chi_0$$

where, we recall,

The equations

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$$\begin{aligned} & -\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f \\ & \chi_t + \partial H(\chi_t) - \Delta \chi + \partial I_{[0,1]}(\chi) \\ & \ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi) \\ & u = u_\Gamma, \partial_n \chi = 0 \quad \text{in } \partial\Omega \\ & \chi(0) = \chi_0 \end{aligned}$$

where, we recall,

- ▶ θ is the (fixed) absolute temperature
- ▶ χ the phase variable
- ▶ u (scalar) small displacement (so that ∇u stands for deformation)

The equations

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$$\begin{aligned} & -\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = f \\ & \chi_t + \partial H(\chi_t) - \Delta \chi + \partial I_{[0,1]}(\chi) \\ & \ni -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi) \\ & u = u_\Gamma, \partial_n \chi = 0 \quad \text{in } \partial\Omega \\ & \chi(0) = \chi_0 \end{aligned}$$

where, we recall,

- ▶ $k(\chi, \theta) = \left(\frac{\chi}{k_1} + \frac{(1-\chi)}{k_2} \right)^{-1} \exp(2E/(\theta - \theta_0)),$
- ▶ $G(\chi, \theta) = C \exp(E/(\theta - \theta_0))(1 - \chi)$

(see [Auricchio, Reali, Stefanelli])

An example

Let us avoid diffusion and viscosity for χ

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Let us avoid diffusion and viscosity for χ

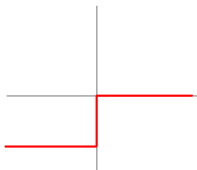
The system is

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = 0$$

$$\partial H(\chi_t) + \partial I_{[0,1]}(\chi)$$

$$\ni -\frac{1}{2}k'(\chi)(\nabla u + G(\chi))^2 + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

where

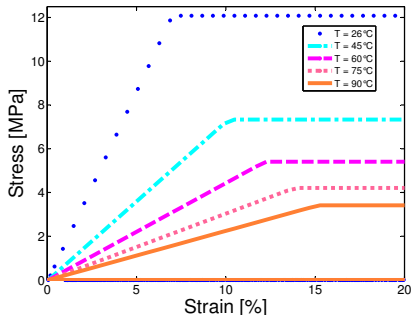


An example

Let us avoid diffusion and viscosity for χ

Start with $\chi = 1$. As $G(1) = 0$ and $k(1) = k_1$ the material is elastic and it is

$$-\operatorname{div} e^{\frac{2E}{\theta - \theta_0}} (k_1 \nabla u - 0) = 0$$



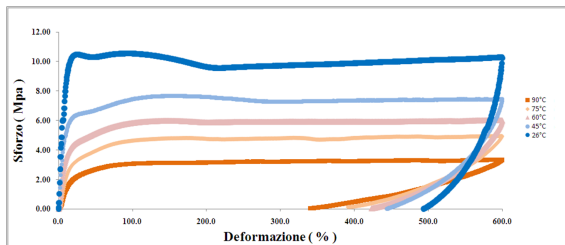
An example

Let us avoid diffusion and viscosity for χ

When **the right hand** side of the evolution equation for χ is $-\eta(\theta)$

$$\partial H(\chi_t) + \partial I_{[0,1]}(\chi) \ni -\frac{1}{2}k'(\chi)\frac{1}{k(\chi)^2}\sigma^2 + \sigma G'(\chi)$$

it is forced $\chi_t \leq 0$ (during this process σ is fixed)



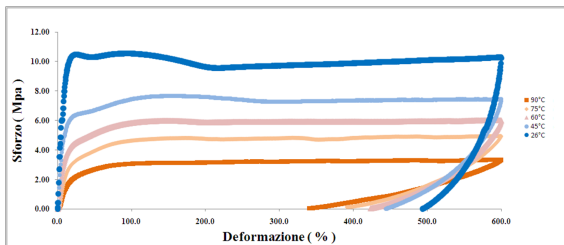
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Let us avoid diffusion and viscosity for χ

When **the right hand** side of the evolution equation for χ is $-\eta(\theta)$

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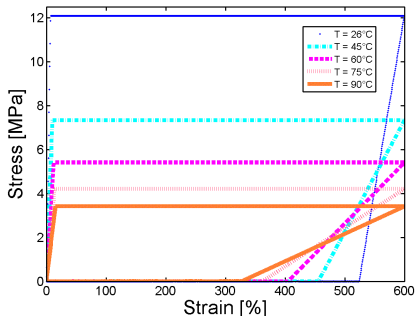


An example

Let us avoid diffusion and viscosity for χ reaching some partial strain corresponding to some χ_s and then unloading, the material evolves in the region $\chi_t = 0$ and $\chi = \chi_s$

$$-\operatorname{div} (k(\chi_s, \theta)(\nabla u - G(\chi_s, \theta))) = 0$$

and the residual strain is at zero stress $G(\chi_s, \theta)/k(\chi_s, \theta)$



The existence result

$$\begin{aligned} -\operatorname{div} (k(\chi)(\nabla u - G(\chi))) &= f(= 0) \\ \chi_t + \zeta - \Delta \chi + \xi &= -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 \\ &\quad + k(\chi)(\nabla u - G(\chi))G'(\chi) \\ \xi &\in \partial I_{[0,1]}(\chi), \quad \zeta \in \partial H(\chi_t) \end{aligned}$$

Theorem There exists a **weak solution** to our problem such that

$$\begin{aligned} u &\in L^\infty(0, T; W^{1,p}(\Omega)), \quad 2 < p < 4, \\ \chi &\in H^1(0, T; L^2(\Omega)) \cap L^\infty(0, T; W^{1,2}(\Omega)) \\ &\quad \cap L^{p/2}(0, T; W^{2,p/2}(\Omega)) \cap L^\infty(Q), \\ \xi &\in L^{p/2}(Q), \\ \zeta &\in L^\infty(Q) \end{aligned}$$

The existence result

Assuming suitable **growth conditions** for ϕ

$$\begin{aligned} -\operatorname{div} (k(\chi)(\nabla u - G(\chi))) &= 0 \\ \zeta - \Delta \chi + \xi &= -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 \\ &\quad + k(\chi)(\nabla u - G(\chi))G'(\chi) \\ \xi &\in \partial I_{[0,1]}(\chi), \quad \zeta \in \partial \phi(\chi_t) \end{aligned}$$

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$$C_1 + C_2|x|^q \leq \phi(x), \quad |\partial \phi(x)| \leq c_1 + c_2|x|^{q-1}$$

Theorem There exists a **weak solution**

$$\begin{aligned} u &\in L^\infty(0, T; W^{1,p}(\Omega)), \quad 2 < p < 4, \quad p = 2 + \varepsilon, \\ \chi &\in W^{1,q}(0, T; L^q(\Omega)) \cap L^\infty(0, T; W^{1,2}(\Omega)) \\ &\cap L^{p/2}(0, T; W^{2,p/2}(\Omega)) \cap L^\infty(Q), \quad 1 < q \leq 1 + \frac{2}{\varepsilon} \\ \xi &\in L^{p/2}(Q), \\ \zeta &\in L^{q'}(Q) \end{aligned}$$

The existence result

Assuming suitable **growth conditions** for ϕ

$$\begin{aligned}-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) &= 0 \\ \zeta - \Delta \chi + \xi &= -\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 \\ &\quad + k(\chi)(\nabla u - G(\chi))G'(\chi) \\ \xi &\in \partial I_{[0,1]}(\chi), \quad \zeta \in \partial \phi(\chi_t)\end{aligned}$$

...everything is easier in 1D! you can directly solve the first equation improving the regularity of ∇u in L^p for any p ... (see also [B., Colli, Dreyer, Gilardi, Schimperna, Sprekels] for a coarsening model)

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The energy estimate

Test by $u_t (= (u - \tilde{u}_r)_t)$ and χ_t , add the resulting equations and get

- ▶ the **energy is bounded**

$$\int_{\Omega} \frac{1}{2} (\chi(t)) (\nabla u - G(\chi))^2(t) + I_{[0,1]}(\chi(t)) + \frac{1}{2} |\nabla \chi(t)|^2 \leq c$$

- ▶ the **dissipation is bounded**

$$\left(\int_0^t \int_{\Omega} \phi(\chi_t) = \right) \int_0^t \int_{\Omega} \frac{1}{2} |\chi_t|^2 + H(\chi_t) \leq c$$

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The estimates

$$u \in L^\infty(0, T; W^{1,2}(\Omega)),$$

$$\chi \in H^1(0, T; L^2(\Omega)) \cap L^\infty(0, T; W^{1,2}(\Omega)) \cap L^\infty(Q),$$

with

$$-\operatorname{div}(k(\chi)(\nabla u - G(\chi))) = 0$$

$$\begin{aligned} \chi_t + \zeta \in L^\infty - \Delta \chi + \xi? &= -\frac{1}{2} k'(\chi)(\nabla u - G(\chi))^2 \in L^1 \\ &+ k(\chi)(\nabla u - G(\chi)) G'(\chi) \end{aligned}$$

The estimates

$$u \in L^\infty(0, T; W^{1,p}(\Omega)), \quad 2 < p < 4$$

$$\chi \in H^1(0, T; L^2(\Omega)) \cap L^\infty(0, T; W^{1,2}(\Omega)) \cap L^\infty(Q),$$

see [Meyers, Zafran, Kunnen, BCDGSS, ...], with

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = 0$$

$$\begin{aligned} \chi_t + \zeta \in L^\infty - \Delta \chi + \xi? &= -\frac{1}{2} k'(\chi)(\nabla u - G(\chi))^2 \in L^{p/2} \\ &+ k(\chi)(\nabla u - G(\chi))G'(\chi) \end{aligned}$$

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then, by a suitable estimates and a comparison

$$-\operatorname{div} (k(\chi)(\nabla u - G(\chi))) = 0$$

$$\chi_t + \zeta \in L^\infty - \Delta \chi + \xi \in L^{p/2} =$$

$$-\frac{1}{2}k'(\chi)(\nabla u - G(\chi))^2 \in L^{p/2} + k(\chi)(\nabla u - G(\chi))G'(\chi)$$

Passage to the limit

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$$\begin{aligned} \chi_t + \partial H(\chi_t) - \Delta \chi + \partial I_{[0,1]}(\chi) = & -\frac{1}{2} k'(\chi)(\nabla u - G(\chi))^2 \\ & + k(\chi)(\nabla u - G(\chi))G'(\chi) \end{aligned}$$

identifying ...

- ▶ by weak and strong compactness + Lebesgue theorem + contracting estimates
- ▶ by monotonicity and lower semicontinuity

Numerical results vs. experiments

The stress-strain curves, tensile tests ...

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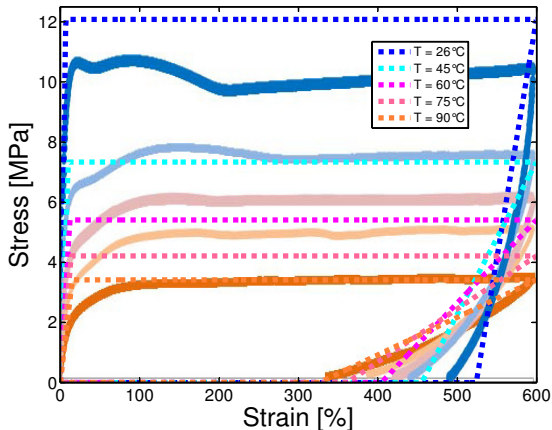
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The stress-strain curves, tensile tests ...
...at different temperature



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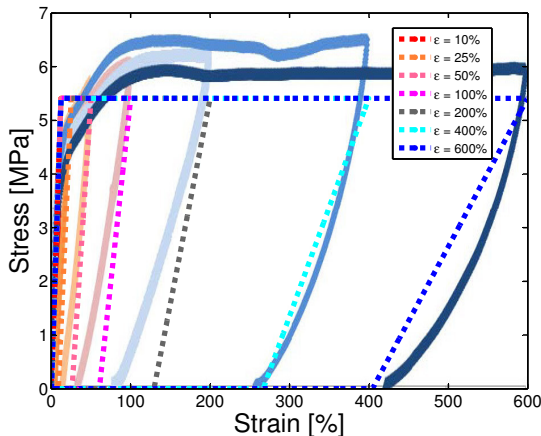
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The stress-strain curves, tensile tests ...
...varying the maximum deformation



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The complex phenomenon

- ▶ Density of semi-crystalline polymers depends on the possibility of creating crystal configurations at each temperature. The phenomenon of crystallization may be represented by some *phase separation* dynamics, we should include in our model.
- ▶ the proportion of oriented chains is normalized with respect to 1, but actually it should depends on the (*variable*) proportion of crystals (with respect to the amorphous phase)
- ▶ *non-isothermal* problem
- ▶ *new constitutive laws* for higher and lower temperature should be included

A first sight: the
PDE

The physical
problem

Semi-crystalline
polymeric materials

The model

The phase variables
Some comments
The existence result

Comparison with
experiments

New perspectives

References

- ▶ B., Castellani, Pachera, Scavello: *A new predictive model for shape memory polymers*, 2013
- ▶ B.: *Existence results for a phase transition model for SMP*, 2013
- ▶ Auricchio, B., Scalet: *Shape memory polymers: constitutive modeling and numerical investigation*, 2013

- ▶ B., Castellani, Pachera, Scavello: *A new predictive model for shape memory polymers*, 2013
- ▶ B.: *Existence results for a phase transition model for SMP*, 2013
- ▶ Auricchio, B., Scalet: *Shape memory polymers: constitutive modeling and numerical investigation*, 2013

Thank you!