① $m = f(0,0) = 1$

$M = f(0, \pm 2) = e^{36}$

$f \longrightarrow g = 4x^2 + 9y^2$

$g|_{\partial K} = 16 + 5y^2 \quad y \in [-2,2]$

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② $f = \cos xy - (1-y^2)x^2 + y$ $\qquad f(-1,0) = 0$

$\frac{\partial f}{\partial y} = -x \sin xy + 2x^2 y + 1$ $\qquad f_y(-1,0) = 1$

$\frac{\partial f}{\partial x} = -y \sin xy - 2x(1-y^2)$ $\qquad f_x(-1,0) = 2$

$g'(-1) = -\frac{f_x(-1,0)}{f_y(-1,0)} = -\frac{2}{1} = -2$

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③ $y = \operatorname{tg} \frac{x^2}{2}$

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④ $l = \int_0^{\pi/2} \sqrt{9 \sin^2 t \cos^2 t (\sin^2 t + \cos^2 t)}\, dt = \int_0^{\pi/2} 3 \sin t \cos t =$

$= \frac{3}{2} \sin^2 t \Big|_0^{\pi/2} = \frac{3}{2}.$

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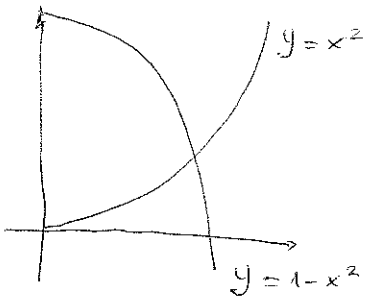
⑥



$D: \begin{cases} 0 \le x \le \frac{\sqrt{2}}{2} \\ x^2 \le y \le 1 - x^2 \end{cases}$

$I = \int_0^{\frac{\sqrt{2}}{2}} dx \int_{x^2}^{1-x^2} \frac{1}{(1+3x^2+y)^2}\, dy$

$= \int_0^{\frac{\sqrt{2}}{2}} \left[\frac{1}{1+3x^2+y} \right]_{y=x^2}^{y=1-x^2} dx = \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{1+4x^2}\, dx - \frac{1}{2} \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{1+x^2}\, dx$

$= \frac{1}{2} \operatorname{arctg} 2x \Big|_0^{\frac{\sqrt{2}}{2}} - \frac{1}{2} \operatorname{arctg} x \Big|_0^{\frac{\sqrt{2}}{2}} = \frac{1}{2} \left(\operatorname{arctg} \sqrt{2} - \operatorname{arctg} \frac{1}{\sqrt{2}} \right)$

(5) $y = e^{-x}(C_1 \cos x + C_2 \sin x) + 3e^{-x}$

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(7) $\frac{\partial f}{\partial x} = \cos(x-y) + e^{x+y}$ $\quad \nabla f(1,1) = (1+e^2, -1+e^2)$

$\frac{\partial f}{\partial y} = -\cos(x-y) + e^{x+y}$ $\quad \langle \nabla f(1,1), (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \rangle = \sqrt{2}e^2$

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(8) $f(\frac{\pi}{2}, 2) = \operatorname{arctg}(\frac{6}{\pi}\frac{\pi}{8} - 2) + \sin\frac{\pi}{2}\cdot 2 = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\frac{\partial f}{\partial x} = \frac{6}{\pi}\frac{1}{1+(\frac{6}{\pi}x - y)^2} + y\cos xy$ $\quad \frac{\partial f}{\partial x}(\frac{\pi}{2}, 2) = \frac{6}{\pi}\frac{1}{8} + 2\cos\pi$

$= \frac{3}{\pi} - 2$

$\frac{\partial f}{\partial y} = -\frac{1}{1+(\frac{6}{\pi}x - y)^2} + x\cos xy$ $\quad \frac{\partial f}{\partial y}(\frac{\pi}{2}, 2) = -\frac{1}{2} + \frac{\pi}{2}\cos\pi$

$= -\frac{1}{2} - \frac{\pi}{2}$

$\pi_{tg} \quad (z - \frac{\pi}{4}) = (\frac{3}{\pi} - 2)(x - \frac{\pi}{2}) + (-\frac{1}{2} - \frac{\pi}{2})(y - 2)$.