$f(x,y) = (x-1)^2 + (y-1)^2$

$\nabla f = 0$ $\quad \begin{cases} 2(x-1) = 0 \\ 2(y-1) = 0 \end{cases} \Rightarrow (1,1)$

Poiché $\quad f(1,1) = 0$

$f(x,y) \geq 0 \quad \forall (x,y) \in \mathbb{R}^2 \quad \Longrightarrow$ (1,1) è punto di minimo assoluto

$(1,1) \in K$

$\boxed{m = 0}$

$\ell_1: \quad x = y^3$
$y \in [0,1]$

$f|_{\ell_1} = (y^3-1)^2 + (y-1)^2 = g_1(y)$

$g_1' = 2(y^3-1)3y^2 + 2(y-1)$

$= 2(y-1)[y^2+y+1]3y^2 + 2(y-1)$

$= 2(y-1)\left[3y^2(y^2+y+1) + 1\right]$

0 ⸻ 1 : $-$

$g(0) = 2$

$g(1) = 1$

$\begin{cases} y = 0 \\ x = 0 \end{cases}$ punto di max

$\ell_2: \quad y = x^3$
$x \in [0,1]$

$f|_{\ell_2} = (x-1)^2 + (x^3-1)^2 = g_2(x)$

$g_2' = 2(x-1)\left[3x^2(x^2+x+1) + 1\right]$

$\begin{cases} x = 0 \\ y = 0 \end{cases}$ punto di max

$\boxed{M = 2}$

---

② $f \in C^\infty(\mathbb{R}^2)$

$f_z = 2(x+z) + 2(y+z) - 12z^5$

$f(0,0,1) = 1 + 1 - 2 = 0$

$f_z(0,0,1) = 2 + 2 - 12 = -\cancel{\ }8$

$f_x = 2(x+y) + 2(x+z) + 2xy^2$ $\qquad f_x(0,0,1) = 2$

$f_y = 2(x+y) + 2(y+z) + 2x^2y$ $\qquad f_y(0,0,1) = 2$

$\pi_{tg} \qquad 2x + 2y - 8(z-1) = 0.$

---

(3)

$$y' = 2y - y^2$$

$$y^{-2}y' = \frac{2}{y} - 1$$

$$\frac{1}{y} = z$$

$$-y^{-2}y' = z'$$

$$y^{-2}y' = -z'$$

$$-z' = 2z - 1$$

$$z' = -2z + 1$$

$$z = e^{-2x}\left[C + \int e^{2x}\right]$$

$$z = Ce^{-2x} + \frac{1}{2}$$

$$z = \frac{De^{-2x} + 1}{2}$$

$$y = \frac{2}{De^{-2x} + 1} \qquad y(0) = 1$$

$$1 = \frac{2}{D+1}$$

$$D + 1 = 2$$

$$D = 1 \qquad y = \frac{2}{1 + e^{-2x}}$$

---

(4)

La serie converge $\forall x \in [-1,1]$

È immediato verificare che c'è convergenza agli estremi (entrambi gli estremi!)

(5) Le serie di potenze sono derivabili termine a termine infinite volte.

Quindi

$$f(x) = \sum_{n=0}^{\infty} (-)^n \frac{x^{n+2}}{(n+1)(n+2)}$$

$$f'(x) = \sum_{n=0}^{\infty} (-)^n \frac{\cancel{(n+2)}\, x^{n+1}}{(n+1)\,\cancel{(n+2)}} = \sum_{n=0}^{\infty} (-)^n \frac{x^{n+1}}{n+1}$$

$$f''(x) = \sum_{n=0}^{\infty} (-)^n x^n = \frac{1}{1+x}$$

Reintegrando

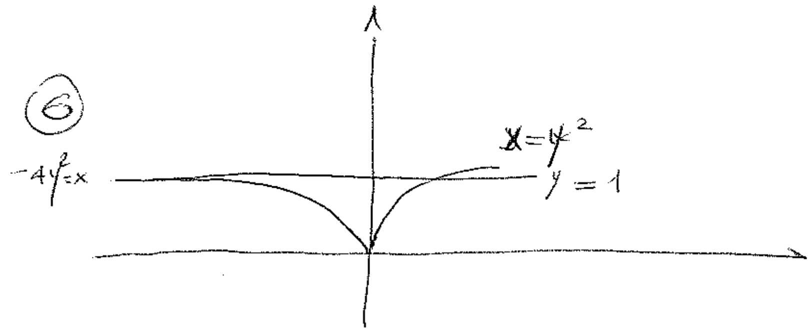
$$f'(x) = \int_0^x \frac{1}{1+t}\,dt = \ln(1+t)\Big|_0^x = \ln(1+x)$$

$$f(x) = \int_0^x \ln(1+t)\,dt = \Big[(1+t)\ln(1+t) - t\Big]_0^x$$

$$= (1+x)\ln(1+x) - x$$

---

(6)



$$\int_D y e^x \,dx\,dy = \int_0^1 y\,dy \int_{-4y^2}^{y^2} e^x\,dx =$$

$$= \int_0^1 y\left(e^{y^2} - e^{-4y^2}\right) dy = \frac{1}{2}\int_0^1 2y\, e^{y^2} dy + \frac{1}{8}\int_0^1 \left(-8y\, e^{-4y^2}\right) dy$$

$$= \frac{1}{2}\, e^{y^2}\Big|_0^1 + \frac{1}{8}\, e^{-4y^2}\Big|_0^1 = \frac{1}{2}(e-1) + \frac{1}{8}\left(e^{-4}-1\right).$$

---

(7) $$r'(t): \begin{cases} x' = 1-2t \\ y' = 2t - 3t^2 \end{cases}$$

Perché $r' \parallel (0,1)$ occorre che $x'(t_0) = 0$, $y'(t_0) \neq 0$

Sarà $t_0 = \frac{1}{2}$

Il punto P ha coordinate $P\left(\frac{1}{4}, \frac{1}{8}\right)$

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(8) $$\int_\ell \frac{y}{x}\, d\sigma_1 = \int_1^2 \frac{x^2}{x}\sqrt{1+4x^2}\, dx = \int_1^2 x\sqrt{1+4x^2}\, dx$$

$$\left(\begin{matrix} 1+4x^2 = t \\ 8x\,dx = dt \\ x\,dx = \frac{1}{8}dt \\ x = 1 \quad t = 5 \\ x = 2 \quad t = 17 \end{matrix}\right) = \frac{1}{8}\int_5^{17} \sqrt{t}\, dt = \frac{1}{8}\,\frac{2}{3}\, t^{3/2} = \frac{1}{12}\left[\sqrt{17^3} - \sqrt{5^3}\right]$$