

Limiti notevoli

$$(1) \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$(2) \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$(3) \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$(4) \quad \lim_{x \rightarrow 0} \frac{\sinh x}{x} = 1$$

$$(5) \quad \lim_{x \rightarrow 0} \frac{\tanh x}{x} = 1$$

$$(6) \quad \lim_{x \rightarrow 0} \frac{\cosh x - 1}{x^2} = \frac{1}{2}$$

$$(7) \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$(8) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e, \quad \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$(9) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{t}{x}\right)^x = e^t, \quad t \in \mathbb{R}$$

$$(10) \quad \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$(11) \quad \lim_{x \rightarrow 1} \frac{\ln x}{x-1} = 1$$

$$(12) \quad \lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a}, \quad a > 0, a \neq 1$$

$$(13) \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$(14) \quad \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a, \quad a > 0, a \neq 1$$

$$(15) \quad \lim_{x \rightarrow +\infty} \frac{e^x}{x^\alpha} = 0, \quad \forall \alpha > 0$$

$$(16) \quad \lim_{x \rightarrow +\infty} \frac{\log_a x}{x^\alpha} = 0^+, \quad \forall \alpha > 0 \quad \text{se } a > 1; \quad \lim_{x \rightarrow +\infty} \frac{\log_a x}{x^\alpha} = 0^-, \quad \forall \alpha > 0 \quad \text{se } 0 < a < 1$$

$$(17) \quad \lim_{x \rightarrow +\infty} x^\alpha a^x = 0, \quad \forall \alpha > 0, 0 < a < 1; \quad \lim_{x \rightarrow -\infty} (-x)^\alpha a^x = 0, \quad \forall \alpha > 0, a > 1$$

$$(18) \quad \lim_{x \rightarrow 0^+} x^\alpha \log_a x = 0^-, \quad \forall \alpha > 0 \quad \text{se } a > 1; \quad \lim_{x \rightarrow 0^+} x^\alpha \log_a x = 0^+, \quad \forall \alpha > 0 \quad \text{se } 0 < a < 1$$