Trasformate di Laplace più significative

Indichiamo con λ l'ascissa di convergenza. Inoltre definiamo

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2}\, dx, \qquad \operatorname{erfc}(t) = \frac{2}{\sqrt{\pi}} \int_t^{+\infty} e^{-x^2}\, dx.$$

(1) $$F(t), \qquad f(s) = \mathcal{L}(F(t); s) = \int_0^{+\infty} e^{-st} f(t)\, dt$$

(2) $$G(t) = F(c\,t),\ c > 0, \qquad g(s) = \frac{1}{c} f(\frac{s}{c}), \qquad \lambda_G = c\,\lambda_F$$

(3) $$G(t) = F(t - t_0),\ t_0 > 0, \qquad g(s) = e^{-t_0 s} f(s), \qquad \lambda_G = \lambda_F$$

(4) $$G(t) = e^{at} F(t),\ a \in \mathbf{C}, \qquad g(s) = f(s-a), \qquad \lambda_G = \lambda_F + \operatorname{Re} a$$

(5) $$G(t) = -t\, F(t), \qquad g(s) = \frac{d}{ds} f(s), \qquad \lambda_G = \lambda_F$$

(6) $$G(t) = F'(t), \qquad g(s) = s f(s) - F(0^+), \qquad \lambda_G = \max\{\lambda_F, \lambda_{F'}\}$$

(7) $$G(t) = \int_0^t F(\tau)\, d\tau, \qquad g(s) = \frac{f(s)}{s}, \qquad \lambda_G = \max\{0, \lambda_F\}$$

(8) $$G(t) = \frac{F(t)}{t}, \qquad g(s) = \int_s^{\infty} f(\tau)\, d\tau, \qquad \lambda_G = \lambda_F$$

(9) $$F(t) = H(t), \qquad f(s) = \frac{1}{s}, \qquad \lambda = 0$$

(10) $$F(t) = H(t) e^{\alpha t},\ \alpha \in \mathbf{C}, \qquad f(s) = \frac{1}{s - \alpha}, \qquad \lambda = \operatorname{Re} \alpha$$

(11) $$F(t) = H(t) \sin \omega t,\ \omega \in \mathbf{R}, \qquad f(s) = \frac{\omega}{s^2 + \omega^2}, \qquad \lambda = 0$$

(12) $$F(t) = H(t)\cos\omega t,\ \omega \in \mathbf{R}, \qquad f(s) = \frac{s}{s^2+\omega^2}, \qquad \lambda = 0$$

(13) $$F(t) = H(t)\sinh\omega t,\ \omega \in \mathbf{R}, \qquad f(s) = \frac{\omega}{s^2-\omega^2}, \qquad \lambda = |\omega|$$

(14) $$F(t) = H(t)\cosh\omega t,\ \omega \in \mathbf{R}, \qquad f(s) = \frac{s}{s^2-\omega^2}, \qquad \lambda = |\omega|$$

(15) $$F(t) = H(t)\,t^n,\ n \in \mathbf{N}, \qquad f(s) = \frac{n!}{s^{n+1}}, \qquad \lambda = 0$$

(16) $$F(t) = H(t)\,t^\alpha,\ \operatorname{Re}\alpha > -1, \qquad f(s) = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}, \qquad \lambda = 0$$

(17) $$F(t) = H(t)\,e^{-t^2}, \qquad f(s) = \frac{\sqrt{\pi}}{2}e^{s^2/4}\mathrm{erfc}(\frac{s}{2}), \qquad \lambda = -\infty$$

(18) $$F(t) = H(t)\,\mathrm{erf}(t), \qquad f(s) = \frac{1}{s}e^{s^2/4}\mathrm{erfc}(\frac{s}{2}), \qquad \lambda = 0$$

(19) $$F(t) = H(t)\,\mathrm{erf}(\sqrt{t}), \qquad f(s) = \frac{1}{s\sqrt{s+1}}, \qquad \lambda = 0$$

(20) $$F(t) = H(t)\ln t, \qquad f(s) = \frac{\Gamma'(1) - \log s}{s}, \qquad \lambda = 0$$

(21) $$F(t) = H(t)\frac{1-e^{-t}}{t}, \qquad f(s) = \ln(1+\frac{1}{s}), \qquad \lambda = 0$$

(22) $$F(t) = H(t)J_0(t), \qquad f(s) = \frac{1}{\sqrt{s^2+1}}, \qquad \lambda = 0$$

(23) $$G(t) = (F_1 * F_2)(t), \qquad g(s) = f_1(s)\cdot f_2(s), \qquad \lambda_G = \max\{\lambda_{F_1}, \lambda_{F_2}\}$$

(24) $$G(t) = H(t)F(t):\ F(t+T) = F(t),\ T>0, \qquad g(s) = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}F(t)\,dt, \qquad \lambda = 0$$