Convoluzioni più significative

(1) $$f(t),\ g(t) \qquad (f*g)(t) = \int_{\mathbf{R}} f(s)\, g(t-s)\, ds = \int_{\mathbf{R}} g(s)\, f(t-s)\, ds$$

(2) $$g_\alpha(t) = e^{-\alpha t^2},\ \alpha > 0, \qquad (g_\alpha * g_\beta)(t) = \frac{\sqrt{\pi}}{\sqrt{\alpha+\beta}}\, e^{-\frac{\alpha\beta}{\alpha+\beta}t^2} = \frac{\sqrt{\pi}}{\sqrt{\alpha+\beta}}\, g_{\frac{\alpha\beta}{\alpha+\beta}}(t)$$

(3) $$f_\alpha(t) = \frac{1}{\alpha^2+t^2},\ \alpha > 0, \qquad (f_\alpha * f_\beta)(t) = \pi\frac{\alpha+\beta}{\alpha\beta}\frac{1}{(\alpha+\beta)^2+t^2} = \pi\frac{\alpha+\beta}{\alpha\beta}\, f_{\alpha+\beta}(t)$$

(4) $$f(t) = H\,\chi_{[-a,a]},\ H \in \mathbf{R},\ a > 0, \qquad (f*f)(t) = H^2\,(2a-|t|)\chi_{[-2a,2a]}(t)$$

(5) $$f(t) = H(-t)\, e^{-\alpha|t|},\ g(t) = H(t)\, e^{-\beta|t|}, \qquad \alpha,\ \beta > 0,$$

$$(f*g)(t) = \frac{1}{\alpha+\beta}(H(-t)\, e^{-\alpha|t|} + H(t)\, e^{-\beta|t|}).$$