ESERCIZI ASSEGNATI NELLA SETTIMANA 18-22/12

$$\int_3^4 e^{-\sqrt{x-2}}\,dx$$

Poniamo

$$-\sqrt{x-2} = t \qquad [\Rightarrow \quad t<0]$$

$$x=3 \qquad t=-1$$

$$x=4 \qquad t=-\sqrt{2}$$

$$x-2=t^2 \qquad x=t^2+2$$

$$dx = 2t\,dt$$

Otteniamo (integrando per parti)

$$\int_{-1}^{-\sqrt{2}} 2t e^t\,dt = \Big[2t\,e^t\Big]_{-1}^{-\sqrt{2}} - 2\int_{-1}^{-\sqrt{2}} e^t\,dt$$

$$= \Big[2te^t - 2e^t\Big]_{-1}^{-\sqrt{2}}$$

$$= -2\sqrt{2}\,e^{-\sqrt{2}} - 2e^{-\sqrt{2}} + 2e^{-1} + 2e^{-1}$$

$$\int_0^{\pi/3} \frac{tgx \ln(4-\cos^2 x)}{\cos x}\,dx = \int_0^{\pi/3} \frac{\sin x \ln(4-\cos^2 x)}{\cos^2 x}\,dx$$

Poniamo

$$\cos x = t \qquad x=0 \qquad t=1$$

$$-\sin x\,dx = dt \qquad x=\pi/3 \qquad t=1/2$$

Otteniamo

$$\int_1^{1/2} -\frac{\ln(4-t^2)}{t^2}\,dt = \int_{1/2}^{1} \frac{\ln(4-t^2)}{t^2}\,dt$$

Integriamo per parti

$$= \left[-\frac{\ln(4-t^2)}{t}\right]_{1/2}^{1} + \int_{1/2}^{1} \frac{1}{t}\,\frac{-2t}{4-t^2}\,dt$$

$$= -\ln 3 + \frac{\ln(15/4)}{1/2} + \int_{1/2}^{1} \frac{2}{t^2-4}\,dt$$

$$\frac{2}{t^2-4} = \frac{A}{t-2} + \frac{B}{t+2}$$

$$A(t+2) + B(t-2) = 2$$

$$t=2 \qquad 4A = 2 \qquad A = 1/2$$

$$t=-2 \qquad -4B = 2 \qquad B = -1/2$$

Dunque

$$\underline{I} = -\ln 3 + 2\ln\frac{15}{4} + \frac{1}{2}\int_{1/2}^{1}\left(\frac{1}{t-2} - \frac{1}{t+2}\right)dt$$

$$= -\ln 3 + 2\ln\frac{15}{4} + \frac{1}{2}\left[\ln\left|\frac{t-2}{t+2}\right|\right]_{1/2}^{1}$$

$$= -\ln 3 + 2\ln\frac{15}{4} + \frac{1}{2}\ln\frac{1}{3} - \frac{1}{2}\ln\frac{3}{5}$$

$$\int_{-1/4}^{1/6} \frac{x}{\sqrt{1-9x^2}} \arccos\sqrt{1-9x^2}\,dx = \int_{-1/4}^{-1/6} \frac{x}{\sqrt{1-9x^2}} \arccos\sqrt{1-9x^2}\,dx$$

$$+ \int_{-1/6}^{1/6} \frac{x}{\sqrt{1-9x^2}} \arccos\sqrt{1-9x^2}\,dx = \int_{-1/4}^{-1/6} \frac{x}{\sqrt{1-9x^2}} \arccos\sqrt{1-9x^2}\,dx$$

Il secondo integrale, infatti, si annulla, perchè la funzione è dispari e l'intervallo di integrazione è simmetrico.

Poniamo

$$\sqrt{1-9x^2} = t \qquad \Rightarrow \qquad t > 0$$

$$x = -\tfrac{1}{4} \qquad t = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$x = -\tfrac{1}{6} \qquad t = \sqrt{1-\frac{9}{36}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$-\frac{18x}{2\sqrt{1-9x^2}}\,dx = dt \qquad \frac{x}{\sqrt{1-9x^2}}\,dx = -\frac{1}{9}\,dt$$

Otteniamo

$$-\int_{\frac{\sqrt{7}}{4}}^{\sqrt{3}/2} \frac{1}{9} \arccos t \, dt = \left[-\frac{1}{9} t \arccos t\right]_{\frac{\sqrt{7}}{4}}^{\frac{\sqrt{3}}{2}} + \frac{1}{9}\int_{\frac{\sqrt{7}}{4}}^{\sqrt{3}/2} \frac{-t}{\sqrt{1-t^2}}\,dt$$

$$= -\frac{\sqrt{3}}{18} \arccos \frac{\sqrt{3}}{2} + \frac{\sqrt{7}}{36} \arccos \frac{\sqrt{7}}{4} + \left[\frac{1}{9}\sqrt{1-t^2}\right]_{\sqrt{7}/4}^{\sqrt{3}/2}$$

$$= -\frac{\sqrt{3}\,\pi}{108} + \frac{\sqrt{7}}{36} \arccos \frac{\sqrt{7}}{4} + \frac{1}{9}\left(\sqrt{1-\frac{3}{4}} - \sqrt{1-\frac{7}{16}}\right)$$

$$= -\frac{\sqrt{3}}{108}\pi + \frac{\sqrt{7}}{36} \arccos \frac{\sqrt{7}}{4} + \frac{1}{9}\left(\frac{1}{2} - \frac{3}{4}\right)$$

$$= -\frac{\sqrt{3}}{108}\pi + \frac{\sqrt{7}}{36} \arccos \frac{\sqrt{7}}{4} - \frac{1}{36}$$

$$\int_{\frac{\sqrt{2}}{2}}^{\sqrt{e^2+1}/2} \frac{16}{9}\, \frac{x \ln(4x^2-1)}{\sqrt[3]{4x^2-1}}\,dx$$

Poniamo

$$4x^2 - 1 = t$$

$$8x\,dx = dt$$

$$x = \frac{\sqrt{2}}{2} \qquad t = 1$$

$$x = \sqrt{e^2+1}/2 \qquad t = e^2$$

Otteniamo

$$\int_1^{e^2} \frac{2}{9} \frac{\ln t}{\sqrt[3]{t}} \, dt = \left[\frac{2}{9} \frac{t^{2/3}}{2/3} \ln t \right]_1^{e^2} +$$

$$- \frac{2}{9} \frac{3}{2} \int_1^{e^2} t^{2/3} \frac{1}{t} \, dt = \frac{1}{3} \left(e^{4/3} \ln e^2 - 1 \ln 1 \right)$$

$$- \frac{1}{3} \left[\frac{t^{2/3}}{2/3} \right]_1^{e^2} = \frac{2}{3} e^{4/3} - \frac{1}{2} e^{4/3} + \frac{1}{2}$$