$$\int_0^{\pi}\left(\frac{x^2}{2\pi}+x\right)\cos(2x)\,dx = \cancel{\left[\left(\frac{x^2}{2\pi}+x\right)\frac{\sin 2x}{2}\right]_0^{\pi}} +$$

$$-\int_0^{\pi}\frac{\sin 2x}{2}\left(\frac{2x}{2\pi}+1\right)dx = \left[\frac{\cos 2x}{4}\left(\frac{x}{\pi}+1\right)\right]_0^{\pi} +$$

$$-\int_0^{\pi}\frac{\cos 2x}{4}\,\frac{1}{\pi}\,dx = \frac{\cos 2\pi}{4}\left(\frac{\pi}{\pi}+1\right) - \frac{\cos 0}{4}\cdot 1$$

$$-\left[\frac{\sin 2x}{8\pi}\right]_0^{\pi} = \frac{1}{4}\cdot 2 - \frac{1}{4} + 0 = \frac{1}{4}$$

$$\int_0^{\pi/2}\sin 2x\,\operatorname{arctg}(3+\cos x)\,dx = -\left[\frac{\cos 2x}{2}\operatorname{arctg}(3+\cos x)\right]_0^{\pi/2}$$

$$+\frac{1}{2}\int_0^{\pi/2}\frac{\cos 2x}{1+(3+\cos x)^2}\cdot(-\sin x)\,dx = -\left[\frac{\cos\pi}{2}\operatorname{arctg} 3\right.$$

$$\left.-\frac{1}{2}\operatorname{arctg} 4\right] - \frac{1}{2}\int_0^{\pi/2}\frac{2\cos^2 x-1}{\cos^2 x+6\cos x+10}\sin x\,dx$$

$$= +\frac{1}{2}\operatorname{arctg} 3 + \frac{1}{2}\operatorname{arctg} 4 + \frac{1}{2}\int_1^0\frac{2t^2-1}{t^2+6t+10}\,dt$$

[avendo effettuato la sostituzione $\cos x = t$] =

$$= \frac{1}{2}(\operatorname{arctg} 4 + \operatorname{arctg} 3) - \frac{1}{2}\int_0^1\frac{2t^2-1}{t^2+6t+10}\,dt$$

Tralasciamo l'ultimo integrale, che è l'integrale (standard) di una funzione razionale fratta. Si noti che il denominatore ha radici complesse coniugate.

$$\int_0^{\sqrt{3}-1} 1 \cdot \operatorname{arctg} \frac{1}{x+1}\, dx = (\text{per parti}) \left[(x+1) \operatorname{arctg} \frac{1}{x+1} \right]_0^{\sqrt{3}-1} +$$

$$- \int_0^{\sqrt{3}-1} \frac{x+1}{1+(x+1)^2}\, dx = \sqrt{3} \operatorname{arctg} \frac{1}{\sqrt{3}} - \operatorname{artg} 1 +$$

$$- \frac{1}{2} \left[\ln\left[1+(x+1)^2\right] \right]_0^{\sqrt{3}-1} = \sqrt{3}\, \frac{\pi}{6} - \frac{\pi}{4} - \frac{1}{2} \ln 4 + \frac{1}{2} \ln 2$$

$$\int_{-\pi/4}^{\pi/2} 4x |\sin 2x|\, dx = \int_{-\pi/4}^{\pi/4} 4x |\sin 2x| + \int_{\pi/4}^{\pi/2} 4x |\sin 2x| dx$$

Il primo integrale è zero, perché si tratta dell'integrale di una funzione dispari su un intervallo simmetrico. Ci riduciamo al secondo

$$\int_{\pi/4}^{\pi/2} 4x |\sin 2x| dx = \int_{\pi/4}^{\pi/2} 4x \sin 2x\, dx$$

infatti $\sin 2x > 0$ $\quad 0 < 2x < \pi \quad 0 < x < \frac{\pi}{2}$

Ora integriamo per parti

$$= \left[-4x \frac{\cos 2x}{2} \right]_{\pi/4}^{\pi/2} + \int_{\pi/4}^{\pi/2} \frac{4}{2} \cos 2x\, dx$$

$$= -\frac{4}{2}\frac{\pi}{2} \cos \pi + \cancel{\frac{4}{2}\frac{\pi}{4} \cos \frac{\pi}{2}} + \left[\sin 2x \right]_{\pi/4}^{\pi/2}$$

$$= \pi - 1$$

$$\int_{5/4}^{7/4} \sqrt{x-1} \arcsin \sqrt{x-1}\, dx = \left[\frac{2}{3}(x-1)^{3/2} \arcsin \sqrt{x-1} \right]_{5/4}^{7/4} +$$

$$- \frac{2}{3} \int_{5/4}^{7/4} \frac{(x-1)^{3/2}}{2\sqrt{x-1}} \frac{1}{\sqrt{1-x+1}}\, dx = \frac{2}{3} \left[\left(\frac{3}{4}\right)^{3/2} \frac{\pi}{3} - \left(\frac{1}{4}\right)^{3/2} \frac{\pi}{6} \right]$$

$$-\frac{2}{3}\int_{5/4}^{7/4} \frac{(x-1)}{2\sqrt{x-1}} \sqrt{\frac{x}{2-x}-1}\, dx$$

Concentriamo su quest'ultimo integrale. Abbiamo

$$+\frac{1}{3}\int_{5/4}^{7/4}\left(\sqrt{2-x}-\frac{1}{\sqrt{2-x}}\right)dx = +\frac{1}{3}\left[\frac{2}{3}(2-x)^{3/2} - 2(2-x)^{1/2}\right]_{5/4}^{7/4}$$

$$\int_1^2 \left[\operatorname{arctg}\frac{t-1}{t} + \sqrt[3]{(t-1)^2}\right]dt = \left[\frac{3}{5}(t-1)^{5/3}\right]_1^2 + \left[t\operatorname{arctg}\frac{t-1}{t}\right]_1^2$$

$$-\int_1^2 t\,\frac{1}{1+\left(\frac{t-1}{t}\right)^2}\left(\frac{t-t+1}{t^2}\right)dt = \frac{3}{5} + 2\operatorname{arctg}\frac{1}{2} +$$

$$-\int_1^2 \frac{t^2\, t}{t^2+(t-1)^2}\,\frac{1}{t^2}\,dt = \frac{3}{5} + 2\operatorname{arctg}\frac{1}{2} - \underbrace{\int_1^2\frac{t}{2t^2-2t+1}\,dt}_{-\frac{1}{4}\int_1^2\frac{4t-2}{2t^2-2t+1}dt - \frac{1}{2}\int_1^2\frac{dt}{2t^2-2t+1}}$$

Concentriamoci sul secondo integrale

$$\int_1^2\frac{1}{2t^2-2t+1}\,dt = \frac{1}{2}\int_1^2\frac{1}{t^2-t+\frac{1}{2}}\,dt = \frac{1}{2}\int_1^2\frac{1}{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}\,dt$$

$$=\frac{1}{2}\int_1^2\frac{1}{\frac{1}{4}\left[\left(\frac{t-1/2}{1/2}\right)^2+1\right]}\,dt = \left[2\operatorname{arctg}\left(\frac{t-1/2}{1/2}\right)dt\right]_1^2$$

$$=\left[2\operatorname{arctg}(2t-1)\right]_1^2 = 2\left[\operatorname{arctg}3 - \operatorname{arctg}1\right]$$

$$\int_0^{\pi/2}\cos x\,\ln(2-\cos^2x)\,dx = \left[\sin x\,\ln(2-\cos^2x)\right]_0^{\pi/2} +$$

$$-\int_0^{\pi/2}\frac{1}{2-\cos^2x}\,2\cos x\sin x\cdot\sin x\,dx$$

$$=\ln 2 - \int_0^{\pi/2}\frac{2\sin^2x}{1+\sin^2x}\cos x\,dx = \ln 2 - \int_0^1\frac{2t^2}{1+t^2}\,dt$$

dopo aver fatto la sostituzione $\sin x = t$.

Tralasciamo la conclusione [redacted]

$$\int_1^2 \frac{e^{2x}}{e^{2x}+2e^x-3}\, dt = \left(\begin{array}{l} e^x = t \\ e^x dx = dt \end{array}\right) = \int_e^{e^2} \frac{t}{t^2+2t-3}\, dt$$

$$= \int_e^{e^2} \left(\frac{A}{t+3} + \frac{B}{t-1} \right) dt$$

$A(t-1) + B(t+3) = t$

$t = 1 \qquad B = 1/4$

$t = -3 \qquad A = +3/4$

$$= \left[+\frac{3}{4} \ln(t+3) + \frac{1}{4} \ln(t-1) \right]_e^{e^2}$$

$$\int_{\pi/2}^{\pi} \frac{x\cos x - \sin x}{x^2}\, dx = \int_{\pi/2}^{\pi} \frac{\cos x}{x}\, dx - \int_{\pi/2}^{\pi} \frac{\sin x}{x^2}\, dx$$

$$= \left[\frac{\sin x}{x} \right]_{\pi/2}^{\pi} + \int_{\pi/2}^{\pi} \frac{\sin x}{x^2}\, dx - \int_{\pi/2}^{\pi} \frac{\sin x}{x^2}\, dx$$

$$= -\frac{2}{\pi}$$

$$\int_0^2 \frac{t^3}{4+3e^{at^4}}\, dt = \left(\begin{array}{l} at^4 = s \\ 4at^3\, dt = ds \end{array}\right) = \frac{1}{4a} \int_0^{16a} \frac{ds}{4+3e^s}$$

$$= \frac{1}{4a} \int_0^{16a} \frac{e^s}{e^s(4+3e^s)}\, ds = \left(\begin{array}{l} e^s = t \\ e^s ds = dt \end{array}\right) = \frac{1}{4a} \int_1^{e^{16a}} \frac{1}{t(4+3t)}\, dt$$

$$= \frac{1}{4a} \int_1^{e^{16a}} \left(\frac{A}{t} + \frac{B}{3t+4} \right) dt$$

$A(3t+4) + Bt = 1$

$t = 0 \qquad A = 1/4$

$t = -\frac{4}{3} \qquad B = -\frac{3}{4}$

$$= \frac{1}{4a} \left[\frac{1}{4} \ln t - \frac{1}{4} \ln(3t+4) \right]_1^{e^{16a}}$$

$$= \frac{1}{16a} \left[16a - \ln(3e^{16a}+4) + \ln 7 \right]$$

Questo è il risultato per $a \neq 0$

[redacted]

Se $a=0$, ci riduciamo a

$$\int_0^2 \frac{t^3}{7}\,dt = \left[\frac{1}{28}t^4\right]_0^2 = \frac{16}{28}.$$

$$\int_{\frac{\sqrt3}{8}}^{a} \frac{1}{x}\frac{1}{\sqrt{1-16x^2}}\,dx = \left(\begin{array}{c} 1-16x^2 = t^2 \\ -32x\,dx = 2t\,dt \end{array}\right) = \int_{\sqrt3/8}^{a} \frac{1}{x^2}\frac{x\,dx}{\sqrt{1-16x^2}}$$

$$= \int_{\frac12}^{1-16a^2} -\frac{1}{32}\frac{2\cdot16\,t}{(1-t^2)\,t}\,dt = -\frac{1}{2}\cdot2\int_{1/2}^{1-16a^2}\frac{1}{1-t^2}\,dt$$

$$= \frac12\cdot2\int_{1/2}^{1-16a^2}\frac{dt}{t^2-1} = \frac12\cdot2\int_{1/2}^{1-16a^2}\left(\frac{A}{t-1}+\frac{B}{t+1}\right)dt$$

$$= \frac14\,2\ln\left|\frac{t-1}{t+1}\right|_{1/2}^{1-16a^2}\,dt$$

$A(t+1)+B(t-1)=1$

$t=1 \qquad A=1/2$

$t=-1 \qquad B=-1/2$

$$a(T) = \int_0^1\left[2-\left|\ln\left(x^2+\frac14\right)\right|\right]dx$$

Studiamo il segno di $\ln\left(x^2+\frac14\right)$. Abbiamo

$\ln\left(x^2+\frac14\right)>0 \qquad x^2+\frac14>1 \qquad x^2>\frac34$

$x>\frac{\sqrt3}{2}$. Pertanto

$$a(T) = \int_{\frac{\sqrt3}{2}}^{1}\left[2-\ln\left(x^2+\frac14\right)\right]dx + \int_0^{\sqrt3/2}\left[2+\ln\left(x^2+\frac14\right)\right]dx$$

Concentriamoci sul calcolo della primitiva di $\ln\left(x^2+\frac14\right)$

Abbiamo

$$\int \ln\left(x^2+\frac14\right)dx = x\ln\left(x^2+\frac14\right) +$$

[redacted]

$$- \int \frac{x \cdot 2x}{x^2+1/4}\, dx = x \ln\left(x^2+\frac{1}{4}\right) - 2\int \frac{x^2+1/4}{x^2+1/4}\, dx$$

$$+ \frac{2}{4}\int \frac{1}{x^2+1/4}\, dx = x \ln\left(x^2+\frac{1}{4}\right) - 2x +$$

$$+ \frac{1}{2}\int \frac{1}{4}\, \frac{1}{1+\left[\frac{\cancel{2}x}{2}\right]^2} = x \ln\left(x^2+\frac{1}{4}\right) - 2x + \frac{1}{4} \operatorname{arctg} \frac{x}{2}$$

Tralasciamo il resto dei calcoli. $\blacksquare$

$$F(x) = \int_{\frac{3}{4}\pi}^{x} \frac{\left(t-\frac{\pi}{2}\right)\left[\cos\left(t-\frac{\pi}{2}\right) + \cos(-t)\right]}{\cos^3 t}\, dt$$

$$= \int_{\frac{3}{4}\pi}^{x} \left(t-\frac{\pi}{2}\right)\left[\frac{\sin t + \cos t}{\cos^3 t}\right] dt$$

$$= \int_{\frac{3}{4}\pi}^{x} \left(t-\frac{\pi}{2}\right)\left(\frac{\sin t}{\cos^3 t} + \frac{1}{\cos^2 t}\, dt\right) =$$

$$= \left[\left(t-\frac{\pi}{2}\right)\left[+\frac{1}{2\cos^2 t} + \operatorname{tg} t\right]\right]_{\frac{3}{4}\pi}^{x} - \int_{\frac{3}{4}\pi}^{x} \left(\frac{1}{2\cos^2 t} + \operatorname{tg} t\right) dt$$

$$= \left(x-\frac{\pi}{2}\right)\left(\frac{1}{2\cos^2 x} + \operatorname{tg} x\right) - \frac{\pi}{4}\left(\frac{1}{2\cos^2 \frac{3}{4}\pi} + \operatorname{tg}\frac{3}{4}\pi\right)$$

$$- \left[\frac{1}{2}\operatorname{tg} t - \ln|\cos t|\right]_{\frac{3}{4}\pi}^{x}$$ $\blacksquare$

Cerchiamo la generica primitiva di $f(x) = \frac{e^{-x}}{\operatorname{ch} x}$

Abbiamo $\int \frac{2e^{-x}}{e^x+e^{-x}}\, dx = \int \frac{2e^{-x}e^x\, dx}{e^{2x}+1} = 2\int \frac{dx}{e^{2x}+1}$

$$= 2\int \frac{e^x}{e^x(1+e^{2x})}\, dx =$$ [redacted]

$$\left(\begin{array}{l} e^x = t \\ e^x dx = dt \end{array}\right) = 2\int \frac{dt}{t(1+t^2)} = 2\int \left(\frac{A}{t} + \frac{Bt+C}{1+t^2}\right) dt$$

$$A + At^2 + Bt^2 + Ct = 1 \implies \begin{array}{l} C = 0 \\ A = 1 \\ B = -1 \end{array}$$

Quindi

$$= 2\left[\ln|t| - \frac{1}{2}\ln(1+t^2)\right]$$

Ritornando alla funzione originale abbiamo

$$\int \frac{e^{-x}}{\operatorname{Ch} x}\, dx = 2\left[\ln e^x - \frac{1}{2}\ln(1+e^{2x}) + C\right]$$

$$= 2\left[x - \frac{1}{2}\ln(1+e^{2x}) + C\right]$$

$$= 2\left[x - \frac{1}{2}\ln[e^{2x}(1+e^{-2x})] + C\right]$$

$$= 2\left[x - \frac{1}{2}\left[2x + \ln(1+e^{-2x})\right] + C\right]$$

$$= 2\left[x - x - \frac{1}{2}\ln(1+e^{-2x})\right] + C\Big]$$

$$= -\frac{2}{2}\ln(1+e^{-2x}) + 2C$$

$$\lim_{x \to +\infty} -\frac{2}{2}\ln(1+e^{-2x}) + 2C$$

$$= \lim_{x \to +\infty} -\frac{2}{2} e^{-2x} + 2C \quad -2C$$

Quindi la primitiva che cerchiamo è

$$F(x) = 2x - \ln(1+e^{2x})$$

[redacted]

$$F(x) = \int_0^x (t+1)\, Ch\,(t^2-2t)\, dt$$

Dobbiamo scrivere $P_2(x,0)$. Utilizziamo gli sviluppi noti

$$Ch\, s = 1 + \frac{s^2}{2} + o(s^2)$$

$$Ch\,(t^2-2t) = 1 + \frac{(t^2-2t)^2}{2} + \dots$$

$$= 1 + \frac{1}{2}\left[4t^2 - 4t^3 + t^4\right] + \dots$$

Quindi in un intorno di $x=0$

$$F(x) = \int_0^x (1+t)\left(1 + 2t^2\right) dt$$

$$= \int_0^x \left(1 + t + 2t^2 + 2t^3\right) dt$$

$$= x + \frac{x^2}{2} + \dots$$

$$P_2(x,0) = x + \frac{x^2}{2}$$

Vogliamo tracciare il grafico di $f : \mathbb{R} \to \mathbb{R}$ definita da

$$f(x) = \int_0^x \left[Ch\,(t^2-t) - 1\right] dt$$

Osserviamo che $g(t) = Ch\,(t^2-t) - 1 \geq 0 \quad \forall t \in \mathbb{R}$.
Perciò $f(x) > 0$ per $x > 0$ e $f(x) < 0$ per $x < 0$. Inoltre $f(0) = 0$.

$$\lim_{x \to +\infty} f(x) = \int_0^{+\infty} \left[Ch\,(t^2-t) - 1\right] dt = +\infty$$

$$\lim_{x \to -\infty} f(x) = \int_0^{-\infty} \left[Ch\,(t^2-t) - 1\right] dt = -\infty$$

[redacted]

$f'(x) = \mathrm{Ch}(x^2 - x) - 1$ per il Teorema fondamentale del Calcolo.

Inoltre $\forall x \in \mathbb{R}$ $f'(x) \geq 0$ e $f'(0) = 0$

Quindi in $x = 0$ la f presenta un flesso a tangente orizzontale

$f''(x) = (2x-1)\,\mathrm{Sh}(x^2 - x)$

$f'' \geq 0$ $(2x-1)(x^2-x) \geq 0$ [Infatti il Sh ha il segno del suo argomento]

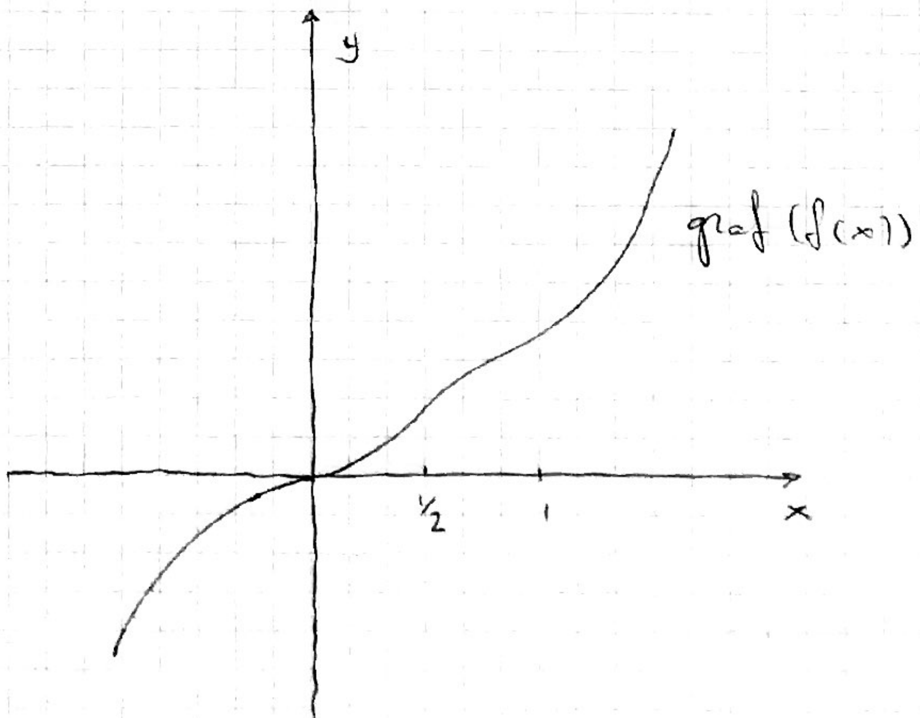
$x(2x-1)(x-1) \geq 0$

	0		$1/2$		1	
$-$		$+$		$-$		$+$

In $x = 0$ flesso ascendente

In $x = \frac{1}{2}$ flesso discendente

In $x = 1$ flesso ascendente



[redacted]

Data $f: \mathbb{R} \to \mathbb{R}$ <u>pari</u> definita da

$$f(x) = \begin{cases} x + 2\arccos\sqrt{x} + \lambda & 0 \le x \le 1 \\ \displaystyle\int_1^x \frac{2e^t}{2t-1}\,dt & x > 1 \end{cases}$$

determinare λ in modo che f sia continua in $\mathbb{R}$.

Abbiamo

$$\lim_{x \to 1^-} f(x) = 1 + 2\arccos 1 + \lambda$$
$$= 1 + 2 \cdot 0 + \lambda = 1 + \lambda$$

$$\lim_{x \to 1^+} f(x) = 0 \qquad \text{(per il T.F.C.)}$$

Quindi dobbiamo richiedere che $1 + \lambda = 0 \Rightarrow \lambda = -1$

Essendo, poi, la f pari

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 2 \cdot \frac{\pi}{2} - 1 = \pi - 1$$

Tralasciamo il grafico di f

Calcolare per $x \to 0$ l'ordine di infinitesimo rispetto a x di

$$f(x) = \int_x^{3x} \left(t - \ln(1 + \sin t)\right) dt$$

È stato svolto a lezione

∎

Calcolare

$$\lim_{x \to 0} \frac{1}{x^3} \int_{\sin x}^{x} \frac{\operatorname{Th} t^2 - \operatorname{tg}(4t)}{t}\,dt$$

Possiamo applicare la formula di De L'Hopital. Al riguardo, combinando il T.F.C. con il Teorema di [redacted] derivazione della funzione composta, si ricordi che

$$\frac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(t)\,dt = f(g_2(x))\,g_2'(x) - f(g_1(x))\,g_1'(x)$$

Pertanto, dobbiamo calcolare

$$\lim_{x\to 0} \frac{\dfrac{(\mathrm{Th}\,x^2 - \mathrm{tg}\,4x)\cdot 1}{x} - \dfrac{\mathrm{Th}(\sin x)^2 - \mathrm{tg}(4\sin x)\cos x}{\sin x}}{3x^2}$$

ma è troppo complicato.

Più semplicemente, osserviamo che

$$\mathrm{Th}\,s = s + \frac{1}{3}s^3 + o(s^3) \qquad \mathrm{tg}\,s = s + \frac{1}{3}s^3 + o(s^3)$$

$$\mathrm{Th}\,t^2 = t^2 + \frac{1}{3}t^6 + o(t^6) \qquad \mathrm{tg}\,4t = 4t + \frac{1}{3}64t^3 + o(t^3)$$

Dunque

$$\frac{\mathrm{Th}\,t^2 - \mathrm{tg}\,4t}{t} = \frac{t^2 + 1/3\,t^6 - 4t - 64/3\,t^3}{t}$$

$$= -4 + t + \dots$$

$$\int_{\sin x}^{x} \frac{\mathrm{Th}\,t^2 - \mathrm{tg}\,4t}{t}\,dt = \int_{\sin x}^{x} (-4 + t)\,dt$$

$$= -4(x - \sin x) + \frac{1}{2}\left[x^2 - (\sin x)^2\right]$$

$$= -4\left[x - \left(x - \frac{x^3}{6}\right)\right] +$$

$$+ \frac{1}{2}\left[x^2 - \left(x - \frac{x^3}{6}\right)^2\right]$$

$$= -4\,\frac{x^3}{6}$$

Pertanto

$$\lim_{x\to 0} \frac{1}{x^3}\int_{\sin x}^{x} \frac{\mathrm{Th}\,t^2 - \mathrm{tg}(4t)}{t} = -\frac{4}{6} = -\frac{2}{3}$$ [redacted]

$$\lim_{x \to 0} \frac{1}{x^3} \int_{x^3}^{x^5} \frac{\ln(1-4t)}{t}\, dt$$

Ricordiamo che

$$\ln(1+s) = s - \frac{s^2}{2} + \frac{s^3}{3} + \dots$$

$$\ln(1-4t) = -4t - \frac{16t^2}{2} - \frac{64t^3}{3} + \dots$$

$$\frac{\ln(1-4t)}{t} = -4 - 8t - \frac{64}{3}t^2 + \dots$$

$$\int_{x^3}^{x^5} \frac{\ln(1-4t)}{t}\, dt = \int_{x^3}^{x^5} \left(-4 - 8t - \frac{64}{3}t^2 + \dots\right) dt$$

$$= -4(x^5 - x^3) - \frac{8}{2}(x^{10} - x^6) + \dots$$

Quindi

$$\lim_{x \to 0} \frac{1}{x^3} \int_{x^3}^{x^5} \frac{\ln(1-4t)}{t}\, dt = \lim_{x \to 0} \frac{4x^3}{x^3} = 4$$

Calcolare

$$\lim_{n \to \infty} \int_{n}^{n+1/n} \sqrt{x^2 + \sin^2 x}\, dx$$

Osserviamo che

$$\sqrt{x^2 + \sin^2 x} = x\sqrt{1 + \left(\frac{\sin x}{x}\right)^2} \quad \text{per } x \geq n$$

Inoltre, per $n \to \infty$, poiché $x > n$, $\frac{\sin x}{x} \longrightarrow 0$ e possiamo concludere che

$$\sqrt{1 + \left(\frac{\sin x}{x}\right)^2} = 1 + \frac{1}{2}\left(\frac{\sin x}{x}\right)^2 + \text{——}$$

ed anche

$$x\sqrt{1 + \left(\frac{\sin x}{x}\right)^2} = x + \frac{1}{2}\frac{\sin^2 x}{x} = x + \dots$$

[redacted] [redacted]

Dunque per $n \to \infty$

$$\int_{n}^{n+\frac{1}{n}} \sqrt{x^2 + \sin^2 x}\, dx = \frac{1}{2}\left[x^2 \right]_{n}^{n+1/n}$$

$$= \frac{1}{2}\left[n^2 + 2 + \frac{1}{n^2} - n^2 \right] = 1$$

e la successione è convergente al valore 1.

[redacted]