ESERCIZI ASSEGNATI IL 12/12/2017

① Stabilire la parte principale per $x \to 0$ di

$$f(x) = (2\sin x - Sh\,2x)^3\,(1 - e^{-x^2})$$

$$2\sin x = 2\left[x - \frac{x^3}{6} + o(x^3)\right]$$

$$Sh\,2x = 2x + \frac{(2x)^3}{6} + o(x^3)$$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2} + o(x^4)$$

Quindi

$$f(x) = \left[\cancel{2x} - \frac{x^3}{3} + o(x^3) \cancel{-2x} - \frac{4x^3}{3} + o(x^3)\right] \cdot$$

$$\cdot \left[\cancel{1} \cancel{-1} + x^2 + o(x^2)\right]$$

$$= \left(-\frac{5}{3}x^3 + o(x^3)\right)\left(x^2 + o(x^2)\right)$$

$$= -\frac{5}{3}x^5 + o(x^5)$$

$$\underset{x \to 0}{\mathcal{P}} f(x) = -\frac{5}{3}x^5$$

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② Calcolare

$$\lim_{x \to 0} \frac{x\,(Ch\,x - \cos x)}{1 + \ln\left(1 + \frac{x}{2}\right) - \sqrt{1+x}}$$

Abbiamo

$$Ch\,x = 1 + \frac{x^2}{2} + \frac{x^4}{4} + o(x^4)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4} + o(x^4)$$

Quindi

$$x(\mathrm{Ch}x - \cos x) = x\left(\cancel{1} + \frac{x^2}{2} + o(x^2) - \cancel{1} + \frac{x^2}{2} + o(x^2)\right)$$

$$= x^3 + o(x^3)$$

D'altro canto

$$\ln\left(1+\frac{x}{2}\right) = \frac{x}{2} - \frac{(x/2)^2}{2} + \frac{(x/2)^3}{3} + o(x^3)$$

$$= \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} + o(x^3)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{x^2}{2} + o(x^2)$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cancel{o(x^2)} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{x^3}{6} +$$
$$+ o(x^3)$$

Dunque

$$1 + \ln\left(1+\frac{x}{2}\right) - \sqrt{1+x} = \cancel{1} + \cancel{\frac{x}{2}} - \cancel{\frac{x^2}{8}} + o(x^3) + \frac{x^3}{24}$$
$$- \cancel{1} - \cancel{\frac{x}{2}} + \cancel{\frac{x^2}{8}} - \frac{1}{16}x^3 + o(x^3)$$

$$= -\frac{1}{48}x^3 + o(x^3)$$

In definitiva

$$\lim_{x\to 0} \frac{x(\mathrm{Ch}x - \cos x)}{1 + \ln\left(1+\frac{x}{2}\right) - \sqrt{1+x}} = \lim_{x\to 0} \frac{x^3 + o(x^3)}{-\frac{1}{48}x^3 + o(x^3)}$$

$$= 48$$

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(3) Calcolare

$$\lim_{x\to 0} \frac{\left(\sqrt{1+2x} - e^x\right)^2}{x(\sin x - \mathrm{Sh}x)}$$

$$\sqrt{1+2x} = 1 + \frac{1}{2}(2x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{(2x)^2}{2} + o(x^2)$$

$$= 1 + x - \frac{1}{2}x^2 + o(x^2)$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$\left(\sqrt{1+2x} - e^x\right)^2 = \left(\cancel{1}+\cancel{x} - \frac{1}{2}x^2 + o(x^2) - \cancel{1} - \cancel{x} - \frac{x^2}{2} + \right.$$

$$\left. + o(x^2)\right)^2 = \left(-x^2 + o(x^2)\right)^2$$

$$= x^4 + o(x^4)$$

$$x(\sin x - \operatorname{Sh} x) = x\left(\cancel{x} - \frac{x^3}{6} + o(x^3) - \cancel{x} - \frac{x^3}{6} + \right.$$

$$\left. + o(x^3)\right) = -\frac{x^4}{3} + o(x^4)$$

Quindi

$$\lim_{x\to 0} \frac{\sqrt{1+2x} - e^x}{x(\sin x - \operatorname{Sh} x)} = \lim_{x\to 0} \frac{x^4 + o(x^4)}{-\frac{x^4}{3} + o(x^4)}$$

$$= -3.$$