- Calcolare i seguenti limiti, **motivando le risposte**.

1. $\lim_{x\to 1} \dfrac{\log^2(x)}{(2x-2)^2}$ $[\frac{1}{4}]$
2. $\lim_{x\to +\infty} x \log\left(\dfrac{x+5}{x-1}\right)$ $[6]$
3. $\lim_{x\to 0} \dfrac{x\sin^2(2x)}{\sin(x^3)}$ $[4]$
4. $\lim_{x\to 0} \dfrac{e^{x^2}-1}{\log(1+2x^2)}$ $[\frac{1}{2}]$
5. $\lim_{x\to +\infty} \sqrt{x^2+3x+2} - |x|$ $[\frac{3}{2}]$
6. $\lim_{x\to +\infty} \sqrt[3]{x+1} - \sqrt[3]{x-1}$ $[0]$
7. $\lim_{x\to +\infty} \dfrac{\sqrt{x}+x}{x}$ $[1]$
8. $\lim_{x\to -\infty} \dfrac{\sqrt{2x^2+3}}{4x+2}$ $[-\frac{\sqrt{2}}{4}]$
9. $\lim_{x\to 1} \dfrac{\cos(\frac{\pi}{2}x)}{1-x}$ $[\frac{\pi}{2}]$
10. $\lim_{x\to e} \dfrac{\log x - 1}{x-e}$ $[\frac{1}{e}]$
11. $\lim_{x\to +\infty} \dfrac{\cos x}{\sqrt{x}}$ $[0]$
12. $\lim_{x\to 0} \sin x \sin\dfrac{1}{x}$ $[0]$
13. $\lim_{x\to +\infty} \left(\dfrac{x-1}{x+3}\right)^{x-2}$ $[e^{-4}]$
14. $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$ $[2]$
15. $\lim_{x\to +\infty} x\sin\dfrac{2}{x}$ $[2]$
16. $\lim_{x\to +\infty} \dfrac{e^x}{e^x-1}$ $[1]$
17. $\lim_{x\to 0} \dfrac{x\tan x}{1-\cos x}$ $[2]$
18. $\lim_{x\to 0} \dfrac{3\arctan x + (1-\cos(2x))\sin^2 x}{27x^4 + 5\sin x}$ $[\frac{3}{5}]$

19. $\lim_{x\to+\infty} \sqrt{x} - 1 + \cos x$ $[+\infty]$

20. $\lim_{x\to 0} \dfrac{x^3 + x^2 \sin x + \sin^2 x}{x^4 + x^3 + x \sin x}$ $[1]$

21. $\lim_{x\to \frac{\pi}{2}} \tan x (e^{\cos x} - 1)$ $[1]$

22. $\lim_{x\to+\infty} \dfrac{\log_2(e^x + 1)}{x + \sin x}$ $[\log_2 e]$

23. $\lim_{x\to 0} \dfrac{1}{x^2} \left(\dfrac{\sqrt{1+3x^2}}{\cos x} - 1 \right)$ $[2]$

24. $\lim_{x\to 2} \dfrac{(\sqrt{x} - \sqrt{2})^2}{x - 2}$ $[0]$

25. $\lim_{x\to 3} \dfrac{\log(3 - \sqrt{x+1})}{3 - x}$ $[+\frac{1}{4}]$

26. $\lim_{x\to 1} \dfrac{e^{\sqrt{x+2}} - e^{\sqrt{3}}}{(x-1)^3}$ $[+\infty]$

27. $\lim_{x\to 0^+} x^{\log x}$ $[+\infty]$

28. $\lim_{x\to 0} \dfrac{1 - \cos(3x) + 7x^3}{\sin^2(5x) + 15x^6}$ $[\frac{9}{50}]$

29. $\lim_{x\to 0} \dfrac{\log(1+x)^3}{\sin(5x) + \sqrt[3]{x^4} \sin x}$ $[\frac{3}{5}]$

30. $\lim_{x\to 0} \dfrac{\log \cos x}{x^2}$ $[-\frac{1}{2}]$

31. $\lim_{x\to 0} \left(\dfrac{1}{x \tan x} - \dfrac{1}{x \sin x} \right)$ $[-\frac{1}{2}]$

32. $\lim_{x\to 0^+} \log_3 x + \dfrac{1}{x}$ $[+\infty]$

33. $\lim_{x\to 0^+} x (\log x)^3$ $[0]$

- Calcolare limiti sinistro e destro di

$$(1)\ \frac{x \sin x}{|x|}; \quad (2)\ \frac{x \cos x}{|x|}; \quad (3)\ |x|^{\frac{1}{x}}; \quad (4)\ (1 + |\sin x|)^{\frac{1}{x}}$$

per $x \to 0$.
Risp. (1): lim sx =lim dx=0; (2) lim sx= -1, lim dx =1; (3) lim sx=$+\infty$, lim dx =0; (4) lim sx =$\frac{1}{e}$, lim dx= e.

- Dire se le seguenti funzioni si prolungano con continuità in 0:

$$2^{\frac{1}{x}} \sin x\,; \quad \arctan \frac{1}{x}\,; \quad \arctan \frac{1}{x^2}\,; \quad x \sin \frac{1}{x}.$$

Risp. [no; no; si; si]