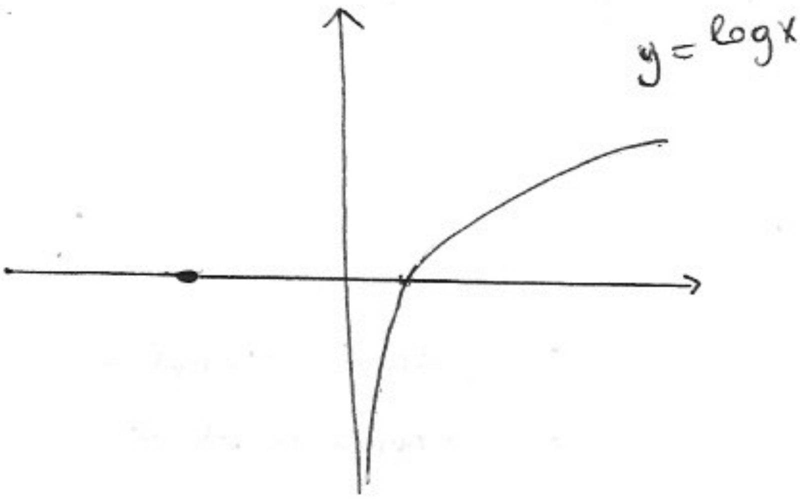
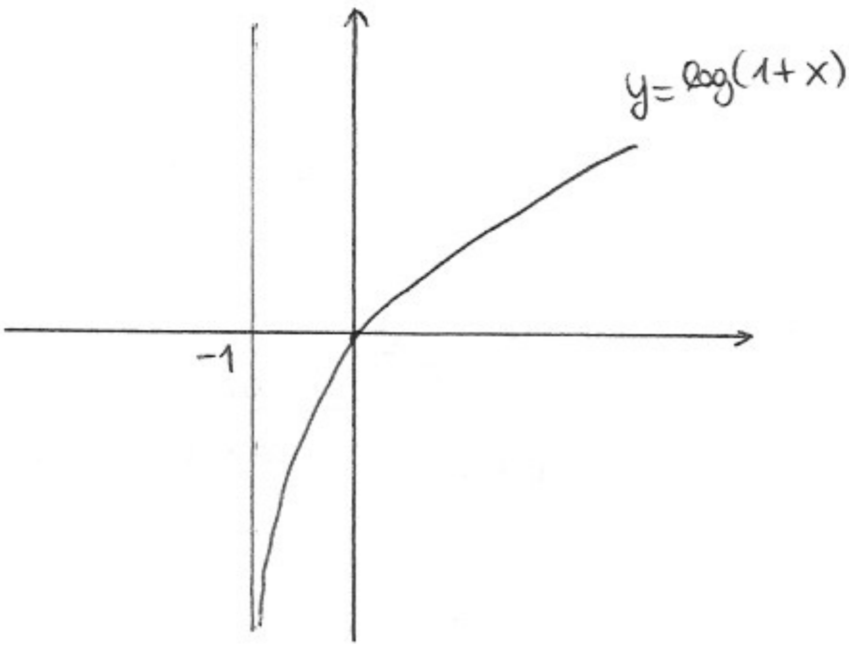
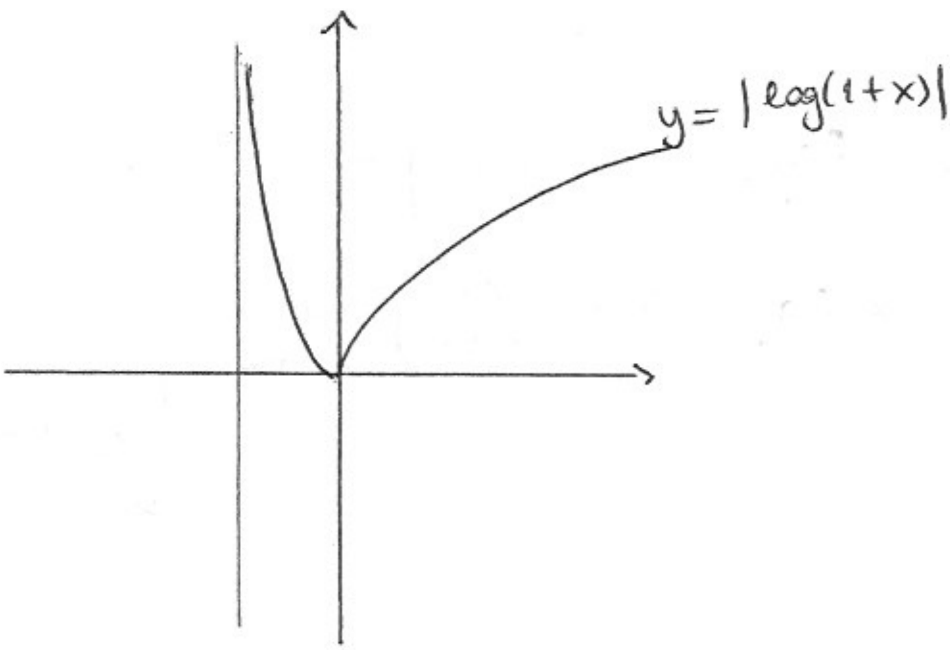
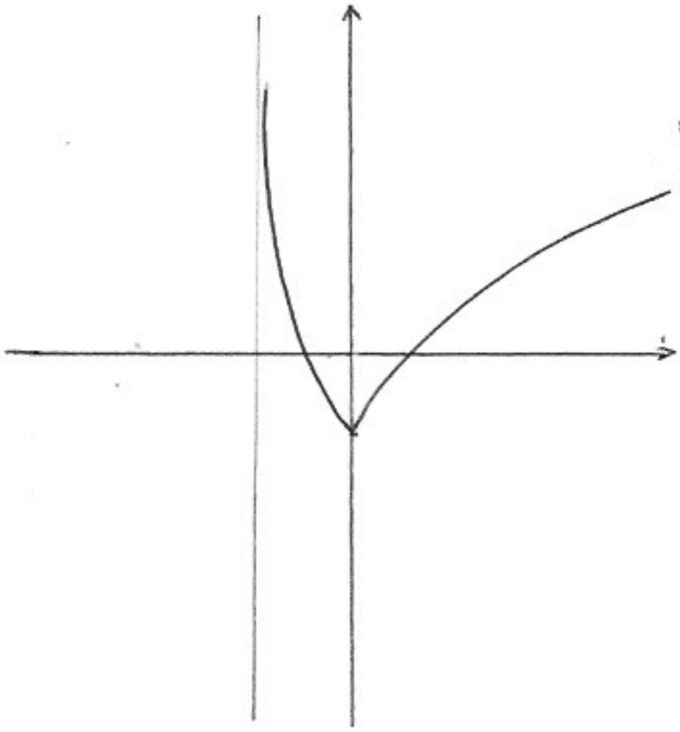
Ex 1 :









$y = |\log(1+x)| - 1$

$x=0$ punto angoloso di minimo assoluto

Ex 2

a) $$\frac{e^{-n}\,\text{arctg}\,n + n!\,n^n}{(n+1)! + e^{\frac{1}{n}}} \sim \frac{n!\,n^n}{(n+1)!} = \frac{n^n}{n+1} \sim \frac{n^n}{n}$$

$$\xrightarrow[n\to+\infty]{} +\infty$$

b) $$x\log\left(\frac{x-1}{x+1}\right) = x\log\left(\frac{x-1}{x+1} - 1 + 1\right) =$$

$$= x\log\left(1 + \frac{-2}{x+1}\right) \underset{x\to+\infty}{\sim} x \cdot \frac{-2}{x+1} \xrightarrow[x\to+\infty]{} -2$$

Ex 3

$$\frac{\sin x - 2x^2}{x^4 - x} \sim \frac{x - 2x^2}{x^4 - x} \sim \frac{x}{-x} \xrightarrow[x \to 0^+]{} -1$$

$$\frac{\sin^2 x \cos\frac{1}{x} - bx}{x} = \frac{\sin^2 x \cos\frac{1}{x}}{x} - b =$$

$$= \underbrace{\frac{\sin x}{x}}_{\substack{\downarrow \\ 1}} \cdot \underbrace{\sin x \cos\frac{1}{x}}_{\substack{\downarrow \\ 0}} - b \longrightarrow -b$$

(limitato · infinitesimo)

f cont. in 0 $\iff$ $f(0^+) = f(0) = f(0^-)$

$\iff$ $a = -1$ e $b = 1$

Ex 4

$$\left| (-1)^n \frac{e^{-n}}{n+1} \right| = \frac{e^{-n}}{n+1} = \frac{1}{e^n (n+1)}$$

App. il cr. del rapporto:

$$\frac{1}{e^{n+1}(n+2)} \cdot e^n (n+1) \longrightarrow \frac{1}{e} < 1$$

$\Rightarrow$ la serie conv. assolut. e semplic.

Ex 5:

$$x^2 - 2x + 8 = 7\left[\left(\frac{x-1}{\sqrt{7}}\right)^2 + 1\right]$$

$$\int \frac{dx}{x^2 - 2x + 8} = \frac{1}{7}\int \frac{dx}{1 + \left(\frac{x-1}{\sqrt{7}}\right)^2} =$$

$$= \frac{1}{\sqrt{7}}\int \frac{\frac{1}{\sqrt{7}}\,dx}{1 + \left(\frac{x-1}{\sqrt{7}}\right)^2} =$$

$$= \frac{1}{\sqrt{7}} \operatorname{arctg}\left(\frac{x-1}{\sqrt{7}}\right) + c$$

$\displaystyle\int_0^{+\infty} \frac{dx}{x^2 - 2x + 8}$ converge in quanto

$f(x) = \dfrac{1}{x^2 - 2x + 8}$ è continua in $[0, +\infty)$

e $f(x) \sim \dfrac{1}{x^2}$ per $x \to +\infty$

Ex 6:

$$(1-i)\, z^5 = 2\bar{z} \longrightarrow$$

$$z = \rho(\cos\theta + i \operatorname{sen}\theta)$$

$$z^5 = \rho^5 (\cos 5\theta + i \operatorname{sen} 5\theta)$$

$$\bar{z} = \rho(\cos(-\theta) + i \operatorname{sen}(-\theta))$$

$$1 - i = \sqrt{2}\left(\cos\frac{7}{4}\pi + i \operatorname{sen}\frac{7}{4}\pi\right)$$

$$\longrightarrow \quad \sqrt{2}\,\rho^5\left[\cos\left(\frac{7}{4}\pi+5\theta\right)+i\,\mathrm{sen}\left(\frac{7}{4}\pi+5\theta\right)\right]=2\rho\left[\cos(-\theta)+ i\,\mathrm{sen}(-\theta)\right]$$

$$\Leftrightarrow \quad \begin{cases} \sqrt{2}\,\rho^5 = 2\rho \\ \frac{7}{4}\pi+5\theta = -\theta+2k\pi \end{cases} \qquad \begin{cases} \rho=0 \quad \vee \quad \rho=\sqrt[4]{\sqrt{2}} \\ \theta=-\frac{7}{24}\pi+\frac{k\pi}{3} \end{cases}$$

$k=0,\dots,5$