Esercizio 1

$$f(x) = x - 2\,\mathrm{arctg}\left(\frac{1}{x}\right)$$

$\mathrm{dom} f = \mathbb{R} \setminus \{0\}$

$f(-x) = -f(x) \quad \forall x \in \mathrm{dom} f \qquad$ (f dispari)

$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = -\pi \qquad \lim_{x \to 0^-} f(x) = \pi$

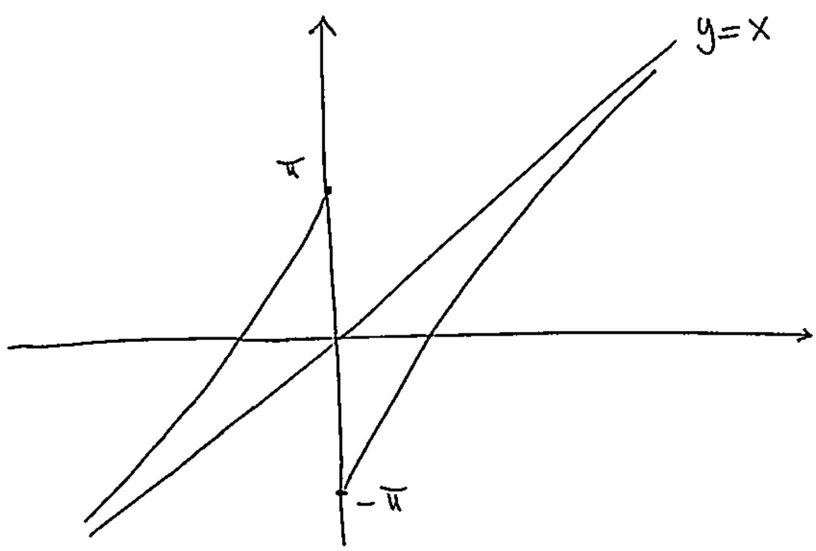
$$\lim_{x \to \pm\infty} \frac{f(x)}{x} = 1$$

$$\lim_{x \to \pm\infty} [f(x) - x] = 0$$

$y = x$ asintoto obliquo

$$f'(x) = 1 - 2\,\frac{-\frac{1}{x^2}}{1 + \frac{1}{x^2}} = 1 + \frac{2}{1+x^2} > 0 \qquad \forall x \in \mathrm{dom} f$$

$\Rightarrow f$ è strett. cresc. in $]-\infty, 0[$ e in $]0, +\infty[$



Esercizio 2

a)

$$\frac{n(n+1)-\sqrt{4n^4+1}}{(n^6+3\log n)\left(\sqrt[3]{1+\frac{1}{n^4}}-1\right)} \sim \frac{-n^2}{n^6\cdot\frac{1}{3}\cdot\frac{1}{n^4}} =$$

$$= -\frac{1}{\frac{1}{3}} = -3 \xrightarrow[n\to+\infty]{} -3$$

b)

$$e^x = 1+x+\frac{x^2}{2}+o(x^2)$$

$$\log(1+2x) = 2x+\frac{1}{2}4x^2+o(x^2)$$

$$e^x-1+\log(1+2x) = x+\frac{x^2}{2}+2x+2x^2+o(x^2)$$

$$= 3x+o(x)$$

$$\sin x = x-\frac{x^3}{6}+o(x^3)$$

$$x-\sin x = \frac{x^3}{6}+o(x^3)$$

$$\frac{\sqrt{x}\,(e^x-1+\log(1+2x))}{\sqrt{x-\sin x}} \sim \frac{\sqrt{x}\cdot 3x}{\sqrt{\frac{x^3}{6}}} = \frac{3\,x^{3/2}}{x^{3/2}}\sqrt{6}$$

$$\xrightarrow[x\to0^+]{} 3\sqrt{6}$$

Esercizio 3

$$f(t) = \frac{t}{(1+t^2)\,\text{arctg}\,t}$$ è positiva e continua in $]0,+\infty[$

per $t \to 0$ $\quad \frac{t}{(1+t^2)\,\text{arctg}\,t} \sim \frac{1}{1+t^2}$

$\text{arctg}\,t \sim t$

$\Rightarrow \lim_{t\to 0^+} f(t) = 1$ $\quad$ f si prolunga con continuità in $t=0$

dunque è integrabile in 0

per $t \to +\infty$ $\quad f(t) \sim \frac{1}{t\,\frac{\pi}{2}} =: g(t)$

$g(t)$ non è integrabile a $+\infty$

$\Rightarrow$ f non è integrabile a $+\infty$

$$\Rightarrow \int_0^{+\infty} \frac{t}{(1+t^2)\,\text{arctg}\,t}\,dt = +\infty$$

$$\lim_{x\to+\infty} \frac{\int_0^x \frac{t}{(1+t^2)\,\text{arctg}\,t}\,dt}{\log x} = \frac{+\infty}{+\infty}$$

De l'Hopital + Teorema fond. del C.I.

$$\frac{\frac{x}{(1+x^2)\operatorname{arctg}x}}{\frac{1}{x}} = \frac{x^2}{(1+x^2)\operatorname{arctg}x} \xrightarrow[x\to+\infty]{} \frac{2}{\pi}$$

Esercizio 4

$$\underbrace{\sum_{n=1}^{+\infty} \frac{n(-1)^n + 4\sin n}{3n+2n^2}}_{S} = \underbrace{\sum_{n=1}^{+\infty} \frac{(-1)^n}{3+2n}}_{S_1} + \underbrace{\sum_{n=1}^{+\infty} \frac{4\sin n}{3n+2n^2}}_{S_2}$$

S_1 converge per il CR. di Leibniz

S_2 converge assolutamente : $\frac{|4\sin n|}{3n+2n^2} \leq \frac{4}{3n+2n^2} \sim$

$\sim \frac{2}{n^2}$

$\sum \frac{2}{n^2}$ converge $\underset{\text{CR. confr.}}{\Longrightarrow}$ $\sum \frac{|4\sin n|}{3n+2n^2}$ converge

$\underset{\text{CR. CONV. ASS.}}{\Longrightarrow}$ $\sum \frac{4\sin n}{3n+2n^2}$ converge

$\Rightarrow$ S converge

Esercizio 5

$$|z|^2 = z\bar{z}$$

$$2z^2\bar{z} - 2z^2 + (1 - i\sqrt{3})\bar{z} - (1 - i\sqrt{3}) = 0$$

$$2z^2(\bar{z} - 1) + (1 - i\sqrt{3})(\bar{z} - 1) = 0$$

$$(2z^2 + 1 - i\sqrt{3})(\bar{z} - 1) = 0$$

$$2z^2 + 1 - i\sqrt{3} = 0 \quad \vee \quad \bar{z} - 1 = 0$$

$2z^2 + 1 - i\sqrt{3} = 0$

$\downarrow$

$$z^2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$= \cos\frac{2}{3}\pi + i\,\text{sen}\frac{2}{3}\pi$$

$\downarrow$

$$z_1 = \cos\frac{\pi}{3} + i\,\text{sen}\frac{\pi}{3}$$

$$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$z_2 = -z_1$$

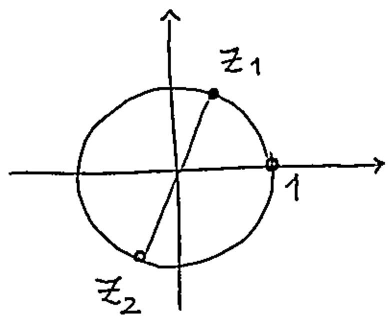
$\bar{z} - 1 = 0$

$\downarrow$

$\bar{z} = 1$

$\downarrow$

$z = 1$



Esercizio 6

$$\lim_{x\to 0^+} x\log x = 0$$

$$\lim_{x\to 0^-} \int_{2x}^{0} \cos(t^2)\, dt = 0$$

$$f(0)=0$$

$\Rightarrow$ f è continua in 0

$$f'(x) = \begin{cases} \log x + 1 & x>0 \\ -\cos(2x)^2 \cdot 2 & x<0 \end{cases}$$

$$\lim_{x\to 0^+} f'(x) = -\infty = f'(0^+)$$

$$\lim_{x\to 0^-} f'(x) = -2 = f'(0^-)$$

f non è derivabile in 0