

Massimi e minimi di funzioni di più variabili

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Esercizio 1 Determinare gli estremi delle seguenti funzioni negli insiemi indicati.

1. $f(x, y) = x^2 + xy + y^2$, $\{(x, y) \in \mathbb{R}^2 \mid |x| \leq 2, |y| \leq 2\}$
2. $f(x, y) = x^2 + y^2$, $\{(x, y) \in \mathbb{R}^2 \mid x^4 + y^4 \leq 1\}$
3. $f(x, y) = \frac{xy}{1+x^2+y^2}$, $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$
4. $f(x, y) = x^2 - xy + y^2 + x + y$, $\{(x, y) \in \mathbb{R}^2 \mid x, y \leq 0, x + y \geq -3\}$
5. $f(x, y) = x^2 - xy + 3y^2 - y$, $\{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1\}$
6. $f(x, y) = x^3 + y^2$, $\{(x, y) \in \mathbb{R}^2 \mid 4x^2 + y^2 \leq 1\}$
7. $f(x, y) = xy$, $\{(x, y) \in \mathbb{R}^2 \mid x^2 - xy + y^2 \leq 1\}$
8. $f(x, y) = xy - y^2 + 3$, $\{(x, y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1 - y^2\}$
9. $f(x, y) = e^x + e^y - 2e^{x-y}$, $\{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1, |y| \leq 1\}$
10. $f(x, y) = (y - x^2)^3$, $\{(x, y) \in \mathbb{R}^2 \mid x + 2 \leq y \leq \sqrt{4 - x^2}\}$
11. $f(x, y) = (1 - x^2 - 4y^2)^2$, $\{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1, |y| \leq 1\}$
12. $f(x, y) = 4x + 4y + 1$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 9\}$
13. $f(x, y) = x^2 + y^2 - xy - x - y$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0, x + y \leq 4\}$
14. $f(x, y) = ye^{-x^2}$, $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 - 2y \leq 0\}$
15. $f(x, y) = x(y^2 + \log(1 + x + y))$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq x, x \leq 1 - y\}$
16. $f(x, y) = x^2y(3 - 2x - 3y)$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0\}$
17. $f(x, y) = x^2 + 3y^2 - x$, T triangolo di vertici $(1, 0), (0, 1), (0, -1)$
18. $f(x, y) = (x - 1)^2 + (y - 1)^2$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, 0 \leq y \leq 4 - 2x\}$
19. $f(x, y, z) = x + y - z^2$, $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq z^2, 0 \leq z \leq 2\}$
20. $f(x, y) = x^2y$, $\{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 1, xy \leq 1\}$

Esercizio 2 Determinare gli estremi delle seguenti funzioni sotto i vincoli indicati a lato.

- (1) $f(x, y) = x^2 + 5y^2 - \frac{1}{2}xy,$ $x^2 + 4y^2 - 4 = 0$
- (2) $f(x, y) = x \log x + y \log y$ $x + y = 1$
- (3) $f(x, y) = (1 + xy)^2$ $x^2 + y^2 = 1$
- (4) $f(x, y) = 4x(x^2 - y^2) - 3x^2 + y^2$ $x^2 - y^2 = 1/4$
- (5) $f(x, y) = x^4 + y^4$ $x + y = -3$
- (6) $f(x, y) = \frac{1}{2}(x^2 + y^2) + \frac{1}{2}$ $x^2 + y^2 + 2x - 2y = -1$
- (7) $f(x, y) = x^2 + y^2$ $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$
- (8) $f(x, y, z) = x + y$ $\begin{cases} z = x^2 + y^2 \\ z = y + 4 \end{cases}$
- (9) $f(x, y, z) = 3x + 4y + 5z - 7$ $\begin{cases} x + y + z = 0 \\ x^2 + y^2 = 1 \end{cases}$
- (10) $f(x, y) = e^x + 3x^2 + y$ $3x^2 + 2x + y = 5$
- (11) $f(x, y) = x^2 - 2x + y^2 - 4y + 2$ $x^2 + y^2 = 2$
- (12) $f(x, y, z) = 3x^2 + 2y^2 + 4z^2$ $2x + 4y - 6z + 5 = 0$
- (13) $f(x, y, z) = x^2 + y^2 + z^2$ $\begin{cases} x + 2y + z - 1 = 0 \\ 2x - y - 3z - 4 = 0 \end{cases}$
- (14) $f(x, y, z) = xy + 2z$ $\begin{cases} x + y + z = 0 \\ x^2 + y^2 + z^2 = 24 \end{cases}$
- (15) $f(x, y, z) = x^2y^3z$ $2x - 3y - z = 0$
- (16) $f(x, y, z) = x^2 + 2y^2 + 3z^2$ $z = 1 - x^2 - y^2, z \geq 0$
- (17) $f(x, y) = 2x + y$ $x^2 - xy + y^2 = 1$
- (18) $f(x, y, z) = xyz$ $x^2 + y^2 + z^2 = 1$