(1') $f(x,y) = e^{-\frac{1}{x^2+y^2}}$ in $\mathbb{R}^2 \setminus \{(0,0)\}$ e $f(0,0)=0$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{\rho\to 0} e^{-\frac{1}{\rho^2}} = 0 = f(0,0)$$

$$f(x,0) = e^{-\frac{1}{x^2}}$$

$$\frac{f(x,0)-f(0,0)}{x} = \frac{1}{x} e^{-\frac{1}{x^2}} \xrightarrow[x\to 0]{} 0 = \frac{\partial f}{\partial x}(0,0)$$

$$f(0,y) = e^{-\frac{1}{y^2}} \qquad \frac{\partial f}{\partial y}(0,0) = 0$$

$$\frac{f(x,y)}{\sqrt{x^2+y^2}} = \frac{1}{\rho} e^{-\frac{1}{\rho^2}} \longrightarrow 0$$

$$\nabla f = e^{-\frac{1}{x^2+y^2}} \left(\frac{2x}{(x^2+y^2)^2}, \frac{2y}{(x^2+y^2)^2} \right)$$

$$\nabla f(1,1) = e^{-\frac{1}{2}} \left(\frac{2}{4}, \frac{2}{4} \right)$$

$$z = e^{-\frac{1}{2}} + e^{-\frac{1}{2}} \cdot \frac{1}{2}(x-1) + \frac{1}{2} e^{-\frac{1}{2}} (y-1)$$

(2') ~~Max e min assoluti esistono per il Th. di Weierstrass.~~

$$\nabla f = (yz, xz, xy)$$

$$\nabla f = (0,0,0) \iff \begin{cases} yz = 0 \\ xz = 0 \\ xy = 0 \end{cases} \qquad \begin{cases} y=0 \vee z=0 \\ \text{—} \\ \text{—} \end{cases}$$

2)

$$\begin{cases} y=0 \\ x=0 \ \vee z=0 \\ 0=0 \end{cases} \quad \vee \quad \begin{cases} z=0 \\ 0=0 \\ x=0 \ \vee \ y=0 \end{cases}$$

asse z, asse x, asse y

$$\begin{cases} y=0 \\ x=0 \end{cases} \vee \begin{cases} y=0 \\ z=0 \end{cases} \vee \begin{cases} z=0 \\ x=0 \end{cases}$$

I pti stazionari sono i pti dei tre assi cartesiani ; ma non sono interni a V

Tuttavia, osserviamo che

$$f(0,y,z)=0$$
$$f(x,0,z)=0 \quad e \quad f(x,y,z) \geq 0 \text{ in } Q$$
$$f(x,y,0)=0$$

$\Rightarrow$ I pti dei piani cartesiani sono tutti pti di min assoluto

3)
$$\begin{cases} t^2 z'' + 3t z' + z = -\dfrac{1}{t} \\ z(e)=1 \\ \lambda z(1) + z'(1) = 0 \end{cases}$$

Eq. Eul. $\quad \alpha(\alpha-1) + 3\alpha + 1 = 0$

$\alpha^2 + 2\alpha + 1 = 0 \quad \alpha = -1$ con molt. 2

$$z(t) = \frac{c_1}{t} + \frac{c_2 \log t}{t} \qquad t>0$$

Soluz particolare: $z(t) = \frac{A}{t}(\log t)^2$

$$z' = -\frac{A}{t^2}(\log t)^2 + \frac{2A\log t}{t^2}$$

$$z'' = \frac{2A}{t^3}(\log t)^2 - \frac{2A\log t}{t^3} - \frac{4A}{t^3}\log t + \frac{2A}{t^3}$$

$$\frac{2A}{t}(\log t)^2 - \frac{6A}{t}\log t + \frac{2A}{t} - \frac{3A}{t}(\log t)^2 + \cancel{\frac{6A}{t}\log t}$$

$$+ \frac{A}{t}(\log t)^2 = \cancel{\frac{A}{t}(\log t)^2} - \frac{1}{t} \iff 2A = -1$$

$$\iff A = -\frac{1}{2}$$

$$z(t) = \frac{c_1}{t} + \frac{c_2 \log t}{t} - \frac{1}{2t}(\log t)^2$$

$$z'(t) = -\frac{c_1}{t^2} - \frac{c_2}{t^2}\log t + \frac{c_2}{t^2} + \frac{1}{2t^2}(\log t)^2 - \frac{\log t}{t^2}$$

$$z(e) = \frac{c_1}{e} + \frac{c_2}{e} - \frac{1}{2e} = 1$$

$$\lambda z(1) + z'(1) = \lambda\left(c_1\right) - \frac{c_1}{1} + c_2 = 0$$

$$\begin{cases} \frac{c_1}{e} + \frac{c_2}{e} = 1 + \frac{1}{2e} \\ (\lambda - 1)c_1 + c_2 = 0 \end{cases}$$

$$\frac{1}{e} - \frac{1}{e}(\lambda - 1)$$

$$\begin{vmatrix} \frac{1}{e} & \frac{1}{e} \\ \lambda - 1 & 1 \end{vmatrix} = \frac{1}{e}(1 - \lambda + 1) = 0$$

$$\iff \lambda = 2$$

$\lambda = 2$ sistema incompatibile $\Rightarrow \nexists$ soluzione

$\lambda \neq 2$ $\exists!$ soluzione

Autovalore $\lambda = 2$

$$\frac{y'}{y^{2/3}} = 2(t+1)$$

$$\frac{dy}{y^{2/3}} = 2(t+1)\,dt$$

$$\frac{y^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} = (t+1)^2 + c$$

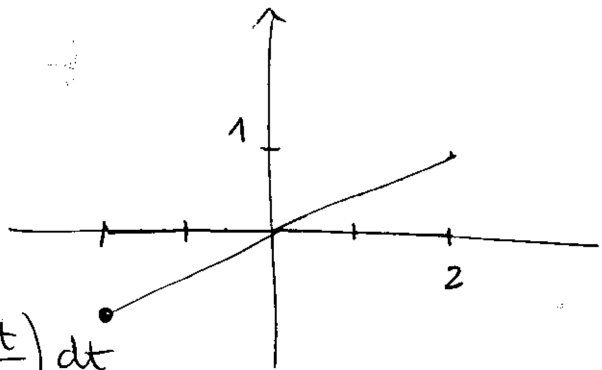
$t = 1$ $$\frac{1}{\frac{1}{3}} = 2^2 + \tilde{c} \quad \Rightarrow \quad c = -1$$

$$y^{1/3} = \frac{1}{3}\left[(t+1)^2 - 1\right]$$

$$y = \left[\frac{1}{3}\left(t^2+2t\right)\right]^3$$

5) Traslo f per averla simmetrica (dispari)

$$\tilde{f} = f - 3 = \frac{1}{2}t$$



$$\tilde{a}_k = 0 \quad \forall k$$

$$\tilde{b}_k = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \tilde{f}(t)\,\mathrm{sen}\left(\frac{2k\pi t}{T}\right)dt$$

$T=4$ $$= \frac{1}{2}\int_{-2}^{2} \tilde{f}(t)\,\mathrm{sen}\left(\frac{k\pi t}{2}\right)dt$$

$\tilde{f}$ dispari $$= \int_0^2 \frac{1}{2}t\ \mathrm{sen}\left(\frac{k\pi t}{2}\right)dt =$$

$$-\frac{1}{2}t\,\frac{\cos\left(\frac{k\pi t}{2}\right)}{\frac{k\pi}{2}}\Bigg|_0^2 + \int_0^2 \frac{1}{2}\,\frac{\cos\left(\frac{k\pi t}{2}\right)}{\frac{k\pi}{2}}\,dt =$$

$$= -\frac{\cos k\pi}{\frac{k\pi}{2}} + 0 + \frac{1}{k\pi}\cdot\underbrace{\frac{\operatorname{sen}\left(\frac{k\pi t}{2}\right)}{\frac{k\pi}{2}}\Bigg|_0^2}_{=0}$$

$$= -\frac{2}{k\pi}(-1)^k = \frac{2}{k\pi}(-1)^{k+1}$$

$$\tilde{f}(x) = \sum_{k=1}^{+\infty} \frac{2}{k\pi}(-1)^{k+1}\operatorname{sen}\left(\frac{k\pi x}{2}\right)$$

$$f(x) = 3 + \sum_{k=1}^{+\infty} \frac{2}{k\pi}(-1)^{k+1}\operatorname{sen}\left(\frac{k\pi x}{2}\right)$$

Parseval

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} \tilde{f}^2(t)\,dt = \frac{T}{2}\left\{\frac{\tilde{a}_0^2}{2} + \sum_{k=1}^{+\infty} \tilde{a}_k^2 + \tilde{b}_k^2\right\}$$

$$\int_{-2}^{2} \frac{1}{4}t^2\,dt = 2\sum_{k=1}^{+\infty}\frac{4}{k^2\pi^2}$$

$$\frac{1}{\cancel{4}_{2}}\cdot\frac{1}{3}\,\cancel{8}\cdot\cancel{2} = \frac{\cancel{8}}{\pi^2}\sum_{k=1}^{+\infty}\frac{1}{k^2}$$

$$\sum_{k=1}^{+\infty}\frac{1}{k^2} = \frac{\pi^2}{6}$$

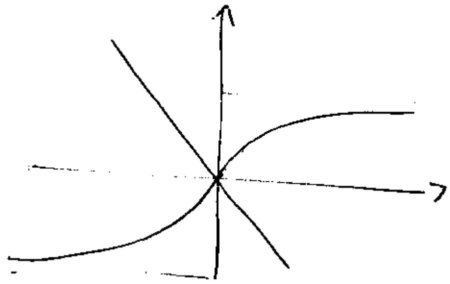
6) $xy^2 + xz - y^3 + \operatorname{arctg} z - 2 = 0$

$(x_0, y_0) = (1, -1)$:

$$\cancel{1} + z + \cancel{1} + \operatorname{arctg} z - \cancel{2} = 0 \Rightarrow z = 0$$

$P = (1, -1, 0)$

$\partial_z : \quad x + \frac{1}{1+z^2}\Big|_P = 1 + 1 \neq 0$



$\Rightarrow \quad \exists! \ \varphi \ U_{(1,-1)} \longrightarrow V_0 \quad$ t.c. $\varphi(1,-1) = 0$

$$f(x, y, \varphi(x,y)) = 0 \quad \forall (x,y) \in U$$

$$\varphi_x = -\frac{f_x}{f_z} \quad ; \quad \varphi_y = -\frac{f_y}{f_z}$$

$f_x = y^2 + z \qquad f_x(P) = 1$

$f_y = 2xy - 3y^2 \qquad f_y(P) = -5$

$\varphi_x(1,-1) = -\frac{1}{2} \quad , \quad \varphi_y(1,-1) = \frac{5}{2}$

$$\frac{\partial \varphi}{\partial v}(1,-1) = \left(-\frac{1}{2}, \frac{5}{2}\right) \cdot \frac{1}{\sqrt{2}}(-1, 1) = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{5}{2}\right) = \frac{3}{\sqrt{2}}$$