Esercizi per l'11/12/19

1. Dire se le seguenti serie a termini positivi convergono o divergono.

   (a) $\displaystyle\sum_{n=0}^{+\infty} \frac{3n^2+1}{n^4+2}$

   (b) $\displaystyle\sum_{n=0}^{+\infty} \sqrt{n}\log\left(1+\frac{1}{n^2}\right)$

   (c) $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{(\log n)^n}$

   (d) $\displaystyle\sum_{n=1}^{+\infty} 1-\cos\left(\frac{1}{n}\right)$

   (e) $\displaystyle\sum_{n=0}^{+\infty} \frac{8^n}{4n^5+10}$

   (f) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n\sqrt[n]{n}}$

   (g) $\displaystyle\sum_{n=0}^{+\infty} e^{-\sqrt{n}}$

   (h) $\displaystyle\sum_{n=0}^{+\infty} e^{-n^2}$

   (i) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{\log(n+1)}$

   (j) $\displaystyle\sum_{n=1}^{+\infty} (n^4-n^2+\log n)\arctan\left(\frac{2}{n^5}\right)$

   (k) $\displaystyle\sum_{n=1}^{+\infty} \frac{n!(2n)!(n+1)^n}{(3n)!}$

2. Studiare la convergenza delle seguenti serie a termini di segno variabile

   (a) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n\frac{\sqrt{n}+(-1)^n}{n}$

   (b) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n}{2n^2+\cos n}$

(c) $\displaystyle\sum_{n=0}^{+\infty}(-1)^n n^2 e^{-n}$

(d) $\displaystyle\sum_{n=1}^{+\infty}\frac{(-1)^{-n}\tan\left(\frac{1}{2n}\right)}{n^2+\sin(n)}$

3. Dire per quali valori del parametro converge ciascuna delle seguenti serie

   (a) $\displaystyle\sum_{n=2}^{+\infty}\frac{n^\alpha}{n^2+\log n}$

   (b) $\displaystyle\sum_{n=1}^{+\infty}(n^2+6)\alpha^n$

   (c) $\displaystyle\sum_{n=1}^{+\infty}\frac{n+1}{n^3\alpha^2+1}$

   (d) $\displaystyle\sum_{n=0}^{+\infty}\frac{(-1)^n}{(1+\alpha^2)^n}$

   (e) $\displaystyle\sum_{n=1}^{+\infty}\frac{(e^{\alpha-3})^n}{\log(n+1)}$

   (f) $\displaystyle\sum_{n=1}^{+\infty}\frac{n^{\alpha+1}}{\arctan\left(\frac{1}{n}\right)+\frac{1}{\sqrt{n}}}$

4. Stabilire se le seguenti affermazioni sono vere o false

   (a) Se $\displaystyle\sum_{n=0}^{+\infty}a_n$ converge allora $\{a_n\}$ è limitata. $\boxed{\text{V}}$ $\boxed{\text{F}}$

   (b) Se $\displaystyle\sum_{n=0}^{+\infty}a_n$ diverge a $+\infty$ allora $\displaystyle\lim_{n\to+\infty}a_n=+\infty$. $\boxed{\text{V}}$ $\boxed{\text{F}}$

   (c) Se $\displaystyle\sum_{n=0}^{+\infty}a_n$ converge allora $\displaystyle\sum_{n=0}^{+\infty}a_n^2$ converge. $\boxed{\text{V}}$ $\boxed{\text{F}}$

   (d) Se $a_n\geq 0$ e $\displaystyle\sum_{n=0}^{+\infty}a_n$ converge allora $\displaystyle\sum_{n=0}^{+\infty}a_n^2$ converge. $\boxed{\text{V}}$ $\boxed{\text{F}}$

   (e) Se $\displaystyle\sum_{n=0}^{+\infty}a_n=1$ allora $\displaystyle\lim_{n\to+\infty}a_n=1$. $\boxed{\text{V}}$ $\boxed{\text{F}}$

   (f) Se $\displaystyle\sum_{n=0}^{+\infty}a_n=1$ allora $\displaystyle\lim_{n\to+\infty}a_n=0$. $\boxed{\text{V}}$ $\boxed{\text{F}}$

(g) Se $a_n \geq 0$, $\sum_{n=0}^{+\infty} a_n$ converge e $\{b_n\}$ è limitata allora $\sum_{n=0}^{+\infty} a_n b_n$ converge. $\boxed{\text{V}}$ $\boxed{\text{F}}$

(h) Se $a_n \leq b_n$ e $\sum_{n=0}^{+\infty} b_n$ converge allora $\sum_{n=0}^{+\infty} a_n$ converge. $\boxed{\text{V}}$ $\boxed{\text{F}}$

(i) Se $\sum_{n=0}^{+\infty} |a_n|$ converge allora $\sum_{n=0}^{+\infty} (-1)^n a_n$ converge. $\boxed{\text{V}}$ $\boxed{\text{F}}$

(j) Se $\sum_{n=0}^{+\infty} (-1)^n a_n$ converge allora $\{a_n\}$ è infinitesima. $\boxed{\text{V}}$ $\boxed{\text{F}}$