Mathematical Models and Methods in Applied Sciences
(2015)

DOI: 10.1142/S0218202515020017

Traffic, crowds, and dynamics of self-organized particles: New trends and challenges

N. Bellomo

Department of Mathematics, Faculty Sciences,
King Abdulaziz University, Jeddah, Saudi Arabia

and

Politecnico Torino, Torino, Italy
nicola.bellomo@polito.it

F. Brezzi

IUSS and IMATI-CNR Via Ferrata 5,
I-27100 Pavia, Italy
brezzi@imati.cnr.it

This brief note is an introduction to the special issue devoted to modeling, qualitative analysis and simulations of vehicular traffic, crowds and swarms, viewed as living, hence complex, systems. The first part focuses on the conceptual difficulties to be tackled when dealing with the most challenging topics of this subject, mainly on interactions and complexity features. The second part provides a brief overview and critical analysis of the contents. Finally, some perspective ideas on conceivable future research objectives are brought to the reader's attention.

Keywords: Traffic; crowds; swarms; emergent behaviors; self-organization; nonlinear interactions; flocking.

This special issue follows various previous ones devoted to the contribution of mathematics to the modeling, qualitative analysis, and simulation of complex living systems, and more precisely of large systems constituted by several living entities interacting in a nonlinear manner. The last special issue on these topics[4] specifically refers to multi-particle systems and to complex systems in general, while the present one focuses on modeling, qualitative analysis, and simulation of traffic, crowds, and self-organized dynamics of large systems of interacting individual entities.

This note is not limited to a technical presentation of some research papers, as it also presents a critical analysis of the challenging research topics under consideration, focusing on conceivable research perspectives. The contents of this issue are occasionally referred to that of the already cited one,[4] in order to understand the development of the research activity in the field as it has been promoted in this journal.

It is useful, before dealing with technical details, to summarize some common features shared by the three systems. A deep analysis of them might contribute to develop new mathematical tools appropriate to capture, as far as possible, the complexity features of the living systems.

- *Vehicular traffic, pedestrian crowds, and animal swarms can be viewed as large systems of living entities. Their interactions are nonlinearly additive, which is one of the main features of complex systems. Hence, it is very difficult to understand and model these systems based on the sole description of the dynamics and interactions of a few individual entities localized in space and time.*[3]
- *The collective overall dynamics is determined by the interactions among the aforementioned living entities, who have the ability to develop strategies based on their own objectives and on these of the other entities. Due to this specific feature, the modeling of individual dynamics based on rules of classical mechanics does not lead in a straightforward way to a mathematical description of collective emerging behaviors.*[5]
- *Traditionally, the modeling approach can be developed at one of the three classical scales: microscopic, namely referred to individuals viewed as nonclassical particles; mesoscopic, where the dependent variable is a probability distribution over the state of microscopic scale; and macroscopic, where the dependent variables, similarly to hydrodynamics, are locally averaged quantities such as number density, linear momentum, and energy. Applied mathematicians are aware that none of the aforementioned scales is exhaustively sufficient for the modeling approach, while multiscale methods are needed not only to obtain the asymptotic limits, which lead to the passage from the low to the higher scale, but also work out modeling approaches, where more than one scale is involved.*[11]
- *The validation of models should be based on their ability to reproduce both phenomena observed in steady conditions and emerging behaviors observed in unsteady dynamics. In the first case the model is required to reproduce empirical data with the needed accuracy, while in the second case models should be able to reproduce the different qualitative dynamics of emerging red behaviors which are generated by similar inputs but are very sensitive to small differences among them. In some cases, large deviations lead to emerging behaviors corresponding to extreme, non-predictable, events, one might, for them, use the term "black swan".*[25]
- *The study of these systems can hopefully lead to a mathematical theory, but might (should) also look at real world applications. Applications often generate interesting mathematical problems. For instance nonlocal interactions, which in traffic appear in junctions, while in crowd dynamics in avoiding obstacles and walls as pedestrians feel at a distance their presence and modify their trajectories accordingly. An important problem is the modeling of social dynamics, which modifies the interaction rules and hence the overall dynamics.*[20]

AQ: Please check the correct word.

The scientific community is well aware that looking at applications generates different hints. On one hand, one looks at problems of interest of our society, such as simulations of crowd dynamics in panic conditions, that can contribute to security of a crowd involved in a disaster,[20] while, on the other hand, new challenging mathematical problems are generated. More in detail living entities have the ability to express specific strategies without the application of any external organizing principle, and this strategy evolves in time and modifies rules of interactions and hence of dynamics. Then models should couple equations modeling motion to other modeling social dynamics. These reasonings suggest that, after the presentation of the contents and out of a critical analysis, we should bring some suggestions toward research targets to the attention of the reader.

Coming now to the contents of the paper, six contributions are numerically distributed over two topics, namely three papers on vehicular traffic and swarms, and three papers focused on collective self-organized motions, including swarms. In details:

- **Traffic and crowds:** Paper 1[1] investigates how the individual behavior of drivers generates queuing dynamics in various physical systems such as pedestrians and vehicles, while paper 2[17] proposes a kinetic theory approach to modeling the flow on networks. A fully discrete kinetic model, with discretization both in the space and velocity, is proposed based on previous papers of the same authors[7,16] and is inserted into the network after an appropriate modeling of junctions. It is worth mentioning that the detailed analysis of Ref. 1 can contribute to a more detailed modeling of junctions. Although the paper is proposed for vehicular traffic, dealing with crowd dynamics on networks appears to be a rather natural generalization. Paper 3[15] is devoted to a deeper understanding of individual based interactions in crowd dynamics. The authors are interested in evacuation processes, jams, and in understanding the so-called Faster is Slower effect, and Stop-and-Go waves. The approach takes into account the granular nature of the flow. Each paper provides an interesting contribution to a deeper understanding of complex phenomena in traffic and crowds. The three papers, viewed as a whole, can contribute to important research developments in the field.
- **Self-organized dynamics:** Paper 4[13] proposes a model of collective behaviors in an annular domain. This paper teaches how specific features of self-organization can be inserted into the classical approach of hydrodynamics. This is an important issue because it focuses on the self-organizing and learning ability of living entities. Papers 5[10] and 6[26] introduce the mathematical approach to control large sparse systems, which is an interesting indication toward new research trends. Moreover, additional challenging topics are treated in these papers. More in detail, Ref. 10 shows how the self-organizing ability generates some optimized patterns, while Ref. 26 introduces the role of leaders in flocking phenomena related to opinion dynamics. Also this group of papers contribute to the search of new ideas and perspectives in the modeling of large systems of interaction living entities.

As we have seen, one of the main focus of all papers has been the transfer of the individual based dynamics to the collective ones. Arguably, this is the most important aspect of the study of self-organized motion.

Let us now look ahead to research perspectives by posing, to ourselves and to the readers, three key questions. The answer to them should address applied mathematicians to some challenging objectives. Interested readers might further develop our answers and hence contribute to mathematical approaches to modeling self-organized dynamics. Accordingly, the following questions, and their answer, are proposed without any claim to be exhaustive:

How should we (mathematicians) derive hints from applications toward fundamental research? Applications pose a variety of new challenging analytic problems such as: dealing with hyperbolic problems with nonlocal interactions[9,14]; derivation of macroscopic hydrodynamic models from the underlying description by mesoscale models[4,12]; transferring the theory of evolutionary games[22,23] into a general theory of differential games; developing a modeling approach on networks.[8,21] Additional ones might be indicated, but the afore mentioned brief list already presents several fruitful interactions between applications and the search of new mathematical tools.

How can the contents of this special issue contribute to the mathematical study of complex systems in different fields of life sciences? A valuable literature already exists on the development of methods from kinetic theory and statistical mechanics to social sciences, documented, among others, in the books,[19,24] while further bibliography is reported in Ref. 5. The general hallmark is that all living systems are complex and exhibit common complexity features.[2] Therefore, it is not unreasonable to develop methods in a certain field of life sciences and transferring them to other fields, which share common features.

Should mathematicians look for a unified mathematical structure for self-organized motion? A rapid answer would be easy, considering that mathematics should always look for a unified structure. However, a constructive reply is far from being at hand. In fact, applied mathematicians are still looking for it, although some attempts can be found in the literature: see Ref. 5 and the references therein. Some reasonings can be motivated by the paper by Gromov,[18] which suggests the search of new mathematical tools suitable to understand the complexity of living systems, and indicates how the quest for new methods should end up with the design of mathematical structures appropriate to constitute the background of any development of interest for the applications.

Needless to say, the answer to these questions can be viewed as a personal view point of the authors of this brief note, while readers' opinion might be different. This is not surprising as the topic of this special issue is not yet fully understood and deserves further investigations. Different research paths can be developed toward the common aim of setting in mathematical equations the complex dynamics of self-organized particles. Definitely, this is a challenging objective, but we hope that the papers of this issue, as well as the perspective ideas of this note, can contribute some trends toward its achievement.

Acknowledgment

The first author gratefully acknowledges support by eVACUATE: a holistic, scenario independent, situation awareness and guidance system for sustaining the Active Evacuation Route for large crowds (Grant Agreement 313161).

References

AQ: Please provide page range in yellow color highlighted reference.

AQ: Ref. not cited in text. Please provide them.

1. C. Arita and A. Schadschneider, Exclusive queueing processes and their application to traffic systems, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.
2. P. Ball, *Why Society is a Complex Matter*: *Meeting Twenty-first Century Challenges with a New Kind of Science* (Springer-Verlag, 2012).
3. M. Ballerini, N. Cabibbo, R. Candelier, A. Cavagna, E. Cisbani, I. Giardina, V. Lecomte, A. Orlandi, G. Parisi, A. Procaccini, M. Viale and V. Zdravkovic, Interaction ruling animal collective behaviour depends on topological rather than metric distance: Evidence from a field study, *Proc. Natl. Acad. Sci. U.S.A.* **105** (2008) 1232–1237.
4. N. Bellomo and F. Brezzi, Complex systems: New challenges with modeling headaches, *Math. Models Methods Appl. Sci.* **24** (2014) 213–219.
5. N. Bellomo, D. Knopoff and J. Soler, On the difficult interplay between life, “complexity”, and mathematical sciences, *Math. Models Methods Appl. Sci.* **23** (2013) 1861–1913.
6. N. Bellomo and J. Soler, On the mathematical theory of the dynamics of swarms viewed as a complex system, *Math. Models Methods Appl. Sci.* **22** (2012) 1140006.
7. A. Bellouquid, E. De Angelis and L. Fermo, Towards the modeling of vehicular traffic as a complex system: A kinetic theory approach, *Math. Models Methods Appl. Sci.* **22** (2012) 1140003.
8. R. Borsche, S. Göttlich, A. Klar, S. Kühn and P. Schillem, The scalar Keller–Segel model on networks, *Math. Models Methods Appl. Sci.* **24** (2014) 221–247.
9. P. L. Buono and R. Eftimie, Analysis of Hopf/Hopf bifurcations in nonlocal hyperbolic models for self-organized aggreagations, *Math. Models Methods Appl. Sci.* **24** (2014) 327–357.
10. M. Caponigro, M. Fornasier, B. Piccoli and E. Trélat, Sparse stabilization and control of alignment models, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.
11. E. Cristiani, B. Piccoli and A. Tosin, *Multiscale Modeling of Pedestrian Dynamics* (Springer, 2014).
12. P. Degond, G. Dimarco and Thi Bich Ngoc Mac, Hydrodynamics of the Kuramoto–Vicsek model of rotating self-propelled particles, *Math. Models Methods Appl. Sci.* **24** (2014) 277–325.
13. P. Degond and H. Yu, Self-organized hydrodynamics in an annular domain: Modal analysis and nonlinear effects, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.
14. R. Eftimie, Hyperbolic and kinetic models for self-organized biological aggregations and movement: A brief review, *J. Math. Biol.* **65** (2012) 35–75.
15. S. Faure and B. Maury, Crowd motion from the granula standpoint, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.

16. L. Fermo and A. Tosin, A fully-discrete-state kinetic theory approach to modeling vehicular traffic, *SIAM J. Appl. Math.* **73** (2013) 1533–1556.
17. L. Fermo and A. Tosin, A fully-discrete-state kinetic theory approach to traffic flow on road networks, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.
18. M. Gromov, In a search for a structure, Part 1: On entropy, July (2012), http://www.ihes.fr/gromov/PDF/structre-serch-entropy-july5-2-2012.pdf.
19. D. Helbing, *Quantitative Sociodynamics. Stochastic Methods and Models of Social Interaction Processes*, 2nd edn. (Springer, 2010).
20. D. Helbing and A. Johansson, Pedestrian crowd and evacuation dynamics, in *Encyclopedia of Complexity and System Scence* (Springer, 2009), pp. 6476–6495.
21. D. Knopoff, On a mathematical theory of complex systems on networks with application to opinion formation, *Math. Models Methods Appl. Sci.* **24** (2014) 405–426.
22. M. A. Nowak, *Evolutionary Dynamics — Exploring the Equations of Life* (Harward Univ. Press, 2006).
23. M. A. Nowak and K. Sigmund, Evolutionary dynamics of biological games, *Science* **303** (2004) 793–799.
24. L. Pareschi and G. Toscani, *Interacting Multiagent Systems*: *Kinetic Equations and Monte Carlo Methods* (Oxford Univ. Press, 2013).
25. N. N. Taleb, *The Black Swan*: *The Impact of the Highly Improbable* (Random House, 2007).
26. S. Wongkaew, M. Caponigro and A. Borzì, On the control through leadership of the Hegselmann–Krause opinion formation model, *Math. Models Methods Appl. Sci.* **25** (2015) xxx.