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using polygonal and polyhedral meshes have been introduced and used since almost half a century. We just recall here the books,^{22,19} and the review papers.^{7,2,20,14,15,17}

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2 N. Bellomo, F. Brezzi & G. Manzini

In recent times the matter received an increasing attention, due to the com-1 bination of several factors that include the convenience in mesh generation, mesh 2 3 deformations, fracture problems, composite materials, topology optimizations, mesh refinements and coarsening, and the like. Recent developments include the evolu-4 tion of Mimetic Finite Differences in the direction of nodal unknowns⁹ or edge 5 unknowns,⁸ the connections with Finite Volumes methods (see e.g. Ref. 13), and, 6 possibly, the recent appearance of the Virtual Element Methods paradigm⁴ that 7 allows a simpler conception, a much easier analysis, and a more straightforward gen-8 eralization of several previous approaches. The present special issue is an attempt 9 to present a variety of different directions that characterize these recent develop-10 ments. Some of the papers included here are mostly review papers. These include 11 the review¹² on Finite Volumes for diffusion equations, the review³ on discontin-12 uous Galerkin methods on very general geometries (including disconnected ele-13 ments), and the guide⁵ on the actual implementation of Virtual Element Methods. 14 Reference 18 presents an overview and some new perspectives on the use of gener-15 alized barycentric coordinates in 2D and 3D diffusion problems and Reference 21 16 addresses the issue related to numerical integration on general polygonal meshes. 17 Both papers discuss (among several other things) the idea of combining classical 18 numerical integration with the exact integration made available (on some suitable 19 part of the stiffness matrix) by the VEM approach for the satisfaction of the patch-20 test for the resulting numerical methods. The importance of an exact satisfaction of 21 the patch-test is underlined in Ref 21. Extensions to nonlinear problems (as quasi-22 linear equations or variational inequalities) are discussed in Ref. 1, while possible 23 additional uses of the free parameters allowed by Mimetic Finite Differences (and 24 Virtual Element Methods) are the object of Ref. 16. 25

In suggesting and collecting papers for this special issue we faced several difficult choices (of topics and authors). We had to neglect relevant applications as *graphics*, as well as very interesting theoretical works, as Ref. 10 on *hp* discontinuous Galerkin methods of polygonal meshes. Finally, the number of references that would have been appropriate for this Preface was indeed much bigger than that present here. We refer instead to the (numerous) references contained in the papers of this special issue.

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