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TRAFFIC, CROWDS, AND SWARMS

N. BELLOMO* and F. BREZZI†

*Department of Mathematics, Politecnico Torino,
Corso Duca degli Abruzzi 24, I-10129 Torino, Italy
nicola.bellomo@polito.it

†Istituto Universitario di Studi Superiori,
Via Luino 4, I-27100 Pavia, Italy
brezzi@imati.cnr.it

The interest in complex systems, namely systems of several individuals interacting in a nonlinear manner, has seen, in recent years, a remarkable increase. Their collective behavior is difficult to be understood and the model is based only on the description of the dynamics of a few individual entities.

This interest is due to a raising awareness that many systems in nature are of this kind and that cannot be successfully modelled by traditional methods used for systems of the inert matter. Moreover, an increasing number of applications in technology, economy, and social sciences resemble such systems, given the high number of elements and the complex interactions among them. See for instance in fields such as transportation and communication networks, or social and economics interchanges.

This special issue is devoted to modelling and simulation of the dynamics of crowds, swarms, and vehicular traffic on roads and networks of roads. The contributions are focused on the study of complex systems, where the overall dynamics is determined by individual interactions, while modelling of individual dynamics does not straightforwardly lead to the mathematical description of the collective dynamics.

As known, different representations and related mathematical modelling approaches can be developed corresponding to the microscopic and macroscopic scales. Moreover, the statistical representation over the microscopic state delivered by suitable generalizations of the mathematical kinetic theory or by master equation methods for multi-agent systems are often used.

It is also well understood that none of the above representations is fully appropriate considering that the number of interacting entities in crowds, swarms, and vehicular traffic is not large enough to justify either the continuum mechanics approximation, namely at the macroscopic scale, or the statistical representation by continuous distribution functions over the microscopic state according to the methods of the kinetic theory.

Moreover, the approach at the microscopic scale involves computational complexity problems related to the number of interacting entities (which may even change in time) that is large enough to generate such a large number of differential equations modelling the individual dynamics that fluctuation errors cannot be avoided.

The modelling at the aforementioned scales can be developed by different approaches. Unfortunately, although in some cases specific models have the ability to provide an acceptable approximation of physical reality, all the above complexity problems do not seem yet to be satisfactorily solved. Possibly, suitable discretizations of the dependent variables into finite volumes, such as that of the cellular automata approach,[14] or discretizations of the phase space in kinetic theory,[11,13] in some cases suitable filters, can be used to average the individual behavior.[16]

An additional, definitely relevant, problem to deal with is modelling the heterogeneous behavior of individuals, that includes their ability to organize interactions and dynamics according to well defined objectives or strategies. This self-organizing ability is not the same for all individuals, and it has to be regarded as a random variable linked to a probability distribution, that may depend locally and that may be modified by interactions at the microscopic level.

Therefore, the modelling approach should include heterogeneity and stochastic features. For instance, the closure of macroscopic equations by material models that include random behaviors. Similarly, when methods of the kinetic theory are applied, interactions should be modelled by stochastic games rather than by deterministic rules. Indeed, interactions do not follow the laws of classical mechanics, but are conditioned by the strategies developed by the interacting individuals.

This special issue is the first of several projects devoted to scientific contributions concerning the above outlined challenging and fascinating topic. A link with a recent series of special issues on cancer modelling,[4] and therein cited bibliography, can be immediately identified. Modelling refers to living systems and involves analytic and computational problems that strongly attract applied mathematicians due to their intrinsic difficulty.

The complex systems dealt with in this issue, namely vehicles on roads or networks of roads, crowds, and swarms, possibly share common features although being characterized by remarkable differences. A common feature is that individuals belonging to the above systems communicate, although in different ways, and have a common strategy. On the other hand, traffic flow is one-directional in one space dimension (or multilane) and over well-defined networks, while the dynamics of crowds and swarms is in two or three space dimensions, either in bounded domains or in the whole space. Crowds may be constrained by particular geometries that generate different aggregation rules. Moreover, in traffic flows, all drivers have approximately the same strategy, which is not consistently modified by outer conditions, while, in crowds, the dynamics of the interactions and the overall strategy is modified according to specific situations, for instance the presence of panic may change them consistently. The modelling approach should capture both analogies and differences. In all cases, the behavior of the system is difficult to understand no matter how simple the behavior of its parts, even though a global pattern or structure certainly occurs.

This special issue includes contribution on traffic flow[5,12]; swarms;[1,8] and crowd dynamics.[2] Collective behaviors can be observed also referring to very special strategies, such as organizations toward criminal actions.[7] Aggregation with special patterns show some analogy with those observed in free crowds although the strategy is very different.

Different scaling and modelling approaches are used: Refs. 2, 5 and 7 develop models derived in the framework of the macroscopic representation in one or two space dimensions, while hydrodynamical models on networks are dealt with in Ref. 12. Methods of statistical mechanics and stochastic differential equations are properly developed to model swarms.[1,8] It is not possible at this stage discussing about the selection of the most appropriate scale, as the research activity in the field, although rapidly progressing, is still at an experimental level and still looks for a unified mathematical approach.

On the other hand, an interesting topic, considered in Refs. 1 and 8, is the derivation of macroscopic equations from the underlying microscopic description. This is a challenging field of applied mathematics traditionally developed for classical or quantum particles, but recently approached also in the case of living systems.[3,9,10,15] Further developments of these methods should provide a deeper understanding of the derivation of models at the macroscopic scale including those[2,5,7] in this special issue.

Hopefully, this issue and the forthcoming ones on analogous complex systems will also contribute to a deeper understanding of the mathematical strategy to be used in modelling large complex living systems.

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