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MATHEMATICS AND COMPLEXITY IN LIFE AND HUMAN SCIENCES

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This special issue is the third of a sequel devoted to the development of mathematical tools for the modeling, qualitative analysis and simulations of complex systems in life and human sciences. Namely of systems of many living individuals interacting in a nonlinear manner.

The collective overall behavior of large systems of interacting individuals is determined by the dynamics of their interactions. On the other hand, a traditional modeling of individual dynamics does not lead in a straightforward way to a mathematical description of the collective emerging behaviors. In particular, it is very difficult to understand and model these systems based on the sole description of the dynamics and interactions of a few individual entities localized in space and time.

The first issue of the sequel was devoted to vehicular traffic, crowds and swarms.[4] The second one[2] was devoted to a variety of different topics such as flocking phenomena,[9] animal epidemics,[12] complex biological phenomena,[15] and social dynamics.[13,14,20]

The contributions to this third special issue are mainly focused on crowds and swarms phenomena for humans and animals. Therefore, the contents appear to be closer to the first issue than the second one. Crowds need to be interpreted in a broad sense, namely not only as an assembly of pedestrians, but also of individuals who aggregate or disaggregate according to specific strategies, for instance aggregation of criminality.[19] An interesting source of information is the material available at the WEB site,[5] and in the report.[6] One of the contributions[17] is in the field of developmental biology, linked however to the other papers by the common objective of analyzing the "distance" between complex entities.

The modeling approach needs a deep understanding of the interaction rules involving individuals in a crowd or in a swarm. These rules generally change according to the type of interacting entities, on their localization, number and physical state. The relevance of this issue is demonstrated (among other facts) by recent theoretical and experimental studies of a team of physicists[1] to understand the dynamics of animal behavior in a swarm. It appears that the strategy developed by each individual depends on the behavior of a fixed number of other animals in the swarm.

Focusing on living systems, definitely the most sophisticated class of complex systems, two main questions can naturally be posed before presenting the contents of this paper:

Do complex living systems exhibit common features?

Are the analytic and computational tools offered by mathematics able to capture, *in the modeling approach*, *the above-mentioned common features*?

A partial answer to the first question can be given borrowing some ideas proposed in the critical analysis presented in Ref. 3. In particular, three main features can be identified, among several others. Specifically:

- *Heterogeneous expression of strategic ability*: Living systems have the ability to express an individual strategy that modifies laws of classical mechanics and, in some cases, generates proliferative and/or destructive processes. This expression is heterogeneously distributed among interacting individuals.
- *Interactions*: Interactions modify the state of the active "particles" according to the strategy they express, on the basis of the space and state distribution of the other "particles". The strategy can be modified by the shape of their distribution. Interactions may not be homogeneous in space, considering that interacting entities can, in some cases, choose different observation paths. Moreover, the distance is not simply geometric, considering that living entities have the ability to identify localization at a great distance and may even privilege it with respect to localization at short distance.
- *Mutations and evolution*: An additional aspect that has to be considered is that, in some cases, the specific properties of living systems evolve in time. This evolution may even correspond to real mutations with substantial modification of the interaction rules related to mutated entities.

Mathematical models should be focused on capturing, at least in part, the above common features. Applied mathematicians should be aware that this effort needs to go beyond the classical tools of statistical and continuum mechanics.

Before trying to answer the second question we shall now give a brief description of the contents of this issue.

Two papers, one by Rodriguez and Bertozzi,[18] and the other by Jones, Brantingham, and Clayes,[16] deal with the modeling of the time and space evolution of criminal behaviors and with the related mathematical problems. Both of them are originated by Ref. 19, which has the merit of having introduced a PDE approach to modeling this new interpretation of crowd dynamics. The first of the two papers deals with the qualitative analysis of mathematical problems generated by application of some of the models under consideration to real situations analysis. The second one develops the approach of Ref. 19 to open systems introducing the role of police actions.

Three papers are related to modeling and mathematical problems in animal swarm dynamics. The paper by Ballerini Cavagna, Cimarelli, Giardina, Parisi, Santagati, Stefanini, and Tavarone[8] provides a sharp analysis of experiments on swarm behavior. It is focused on understanding how birds in a swarm develop their dynamics by taking advantage of the surrounding neighbors. It appears that the individual dynamics is nonlinearly related to that of a fixed number of neighboring individuals, independently of their distance as far as it is not too large. This paper offers to applied mathematicians various challenging hints to develop new modeling approaches, which go beyond models based on linear mean field interaction rules. On the other hand the paper by Carrillo, Klar, Martin, and Tiwari[7] mainly deals with the modeling of swarms undergoing mean field interaction rules, but also subject to a roosting force. They show that this last action generates emerging behaviors that are not depicted by the model in the absence of a force field.

Multiscale issues are dealt with in the paper by Degond and Tong Yang,[11] focused on the analysis of the links between the microscopic and the macroscopic descriptions. This paper follows a previous one by Degond and Motsch,[10] presenting a higher-order derivation of the large scale model based on the underlying description at the small scale.

All the contributions mentioned above do not consider the evolutionary aspect of several complex living systems, where interaction rules evolve in time. This aspect generates difficult mathematical problems related to the qualitative and computational analysis of systems of equations that change type and, in some cases also change in number, as documented in Ref. 12 published in the preceding issue on complex systems. A technical aspect somehow related to evolution is considered here in the paper by Pompei, Caglioti, Loreto and Tria[17] dealing with distance-based phylogenetic algorithms.

Looking back to our second question (in the beginning of this Preface), and revisiting the common features that we identified above, we can now state that some of them have been considered in the papers of this special issue. Moreover, we might also state that, at a general level, modeling them by traditional methods does not yield a satisfactory account of all the relevant phenomena that are observed in nature. Therefore, mathematicians and physicists are challenged by the need to look for new methodological approaches to complexity. For instance, applied mathematicians should face, in the modeling approach, the characteristics of living systems where interacting entities have the ability to organize specific strategies that depend not only on the state of the entities, but also to their number and localization. Additional difficulty is generated by the heterogeneous distribution of their ability to express a strategy, which, in some cases evolves in time.

These issues are also motivated by the fact that an increasing number of applications in technology, economy and social sciences can borrow techniques from complex systems with great benefit. Our hope is that this issue can contribute to a deeper understanding of the mathematical strategy to be used in modeling large complex living systems. Possibly by new mathematical tools.

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