(1) Show that the Korteveg-de-Vries equation

$$\frac{\partial u}{\partial t} + \beta \frac{\partial u}{\partial x} + \alpha \frac{\partial^3 u}{\partial x^3} = 0$$

admits the closed form solution $u(x,t) = a\cos(kx - \omega t)$.
(a,α, and β given; ω to be properly chosen, namely $\omega = k\beta - \omega t$)

(2) Classify the following PDE and determine its characteristic lines

$$x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$

(3) Write a weak formulation for the following equation, describing the vertical displacement of a cable of unit length subjected to the load w (T denotes the tension and k the elastic coefficient)

$$-u''(x) + \frac{k}{T} u(x) = \frac{w(x)}{T}$$

(4) Study the consistency order of the following one dimensional approximation of the first derivative u_i'

$$\frac{-11u_i + 18u_{i+1} - 9u_{i+1} + 2u_{i+3}}{6h}$$

(5) Compute explicitly the stiffness matrix corresponding to the bilinear form $a(u,v) = \int_0^1 u'(x)v'(x)\,dx$ and associated to a finite element discretization by means of piecewise linear functions on a uniform mesh.