**LES CAHIERS DU NUMÉRIQUE**

**The APUPA bell curve**

***Ranganathan's visual pattern for knowledge organization***

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Among many clever innovations, the pioneer of faceted classifications, S.R. Ranganathan, introduced a structural model called APUPA (Alien-Penumbral-Umbral-Penumbral-Alien). This model can be visualized in the form of a bell curve, like that named after mathematician C.F. Gauss.

APUPA is commonly cited as an ideal shape resulting from the arrangement of works according to the principles of faceted classification, which should enable a reader browsing shelves to visually find "his book" as located in the middle of a sequence of related subjects on both its left and its right. However, we show how this is but one application of a more general model of thought recurring throughout Ranganathan's works. Main classes order of Colon Classification, and anteriorizing common isolates, are two more features where the bell-curve model can be observed.

We analyze the presence of this concept in Ranganathan's work through excerpts from a variety of his published books. The concept is compared with models adopted in the work of other classificationists, and with various ideas available in scientific literature, including those on human visual perception, Bernoulli numbers and the mathematical theory of communication. It can be concluded that the APUPA model is yet another facet of the deep originality of Ranganathan's complex and rich classificatory thought.

**1. Introduction**

Knowledge organization systems (KOS) have always been developed according to some structuring principles, which are often visualized by such metaphors as ladders, circles, trees, webs etc. (e.g. Rossi, 2000; Weingart, 2013).

Faceted classifications are no exception to this, as they are structured according to frames of facets in a fixed citation order, each in turn divided into tree-like hierarchies (Gnoli, 2010).

However, an additional structuring principle can be identified in the prototype of all faceted classifications, that is, S.R. Ranganathan’s Colon Classification. This principle can be visualized as a bell-curve pattern, and corresponds to what Ranganathan describes as the APUPA model. It thus seems to have an essentially geometrical nature, connected to Gaussian curves, that can have been inspired to Ranganathan by his background as a former mathematician.

Recent literature has considered APUPA in relation with such topics as collection development and library cooperation (Shakunthala, 1993) or citation analysis (Pichappan, 1996). A careful examination of Ranganathan’s works reveals that, far from being a simple device introduced in specific points to solve practical problems, the APUPA bell curve is an original conceptual model applied consistently throughout his work.

In the next sections we will illustrate and discuss its application in the order of Colon main classes, in anteriorizing common isolates that can be specified for each class, and in Ranganathan’s conception of shelf browsing. We will then suggest connections of this with some scientific theories that were available at Ranganathan's time.

**2. APUPA in Ranganathan’s works**

In Ranganathan’s words, APUPA is an “arrangement complying with the pattern Alien, Penumbral, Umbral, Penumbral, Alien, that is, starting with the class of the isolate or the array-isolate of primary interest to the reader at the moment on either side of it following successively the classes or the isolates, the array-isolates of progressively decreasing interest to him, until they fade out into classes, or isolates, or array isolates not of interest to him” (Ranganathan, 1965, 290).

In other words, books on a focus subject of interest are assumed to be shelved in the middle of a sequence of books of related subjects, with relevance gradually decreasing both on the left and on the right of the focal point.

The first work in which Ranganathan discusses the APUPA idea seems to be *Classification and Communication* (1951), a book devoted to determine the domain of classification in communication.

Nevertheless, the core theme of arrangement of knowledge units is present in Ranganathan’s works since his master and seminal work *The five laws of Library Science*. Indeed, arrangement and its kinds have a direct relevance for the Fourth Law: while an alphabetic arrangement on the shelves is viewed as counterproductive, “the Fourth Law [is led] to press the claims of the classified mode of arrangement. We have seen that the Second and Third Laws also prefer such an arrangement. Each law approaches the problem of shelf-arrangement from a different angle; but fortunately, their conclusions are all concurrent.” (Ranganathan, 1931, 343–344).

Further, Ranganathan highlights that “the Fourth Law would throw some further light on the arrangement. In the first place, it would recommend that the classes accommodation in adjacent shelves should have the greatest possible affinity. A reader that is primarily interested in philosophy is sure to have some interest in religion and psychology”. Further, he adds that “apart from relative position, the absolute position of the classes also will have to be determined in conformity to the Fourth Law. The class of books that is most in demand should be put on the nearest shelves of the stack-room and the class that is least popular should, ordinarily, be put at the farthest end” (Ranganathan, 1931, 343).

Reflections on arrangement are pervasive in Ranganathan’s work, as this is a core issue in library science. A few years later, in its *Theory of library catalogue*, he writes: “Arrangement is the very essence of a catalogue”. While discussing the possible alphabetical arrangement of 18 entries, he records that “this would indeed seem as simple as *a b c* but it is not really so” (Ranganathan, 1938, 354) and that an arrangement based on subject classes should be applied also to entries with the very same entry word. For example, entries with the same entry word ‘Madras’ should be listed in classes for Government Corporate Entries, for Specific Subject Entries involving the whole province Madras, for Municipality Entries, etc.

From *Classification and Communication* on, APUPA is a recurrent theme in all Ranganathan’s major works: it appears in *Reference Service* (1961), in *Classified Catalogue Code with additional rules for Dictionary catalogue code* (1964), in *A descriptive account of Colon Classification* (1965), and in *Prolegomena to Library Classification* (1967); such continuous presence shows the relevance of this concept among his theoretical ideas.

Communication of intellectual content is the main goal of any bibliographic system. Nevertheless, Ranganathan records that “communication does not get completed unless facilities are provided for organizing the records and keeping them arranged in a *helpful order*, picked out expeditiously, exhaustively, and exactly, that is without any unwanted record also coming along and hampering consumption”[[1]](#footnote-0) (Ranganathan, 1951, 227). Unwanted – or irrelevant – records cause distraction, and are equivalent to noise. So arrangement works as an information retrieval system: it must give access to, and only to, relevant records.

The issue is to define a clear line between relevant and irrelevant records: in fact “various degrees of intensity are possible in the relevance and irrelevance of records” (Ranganathan, 1951, 227), due to objective and subjective evaluation elements. A record which is relevant for a user could be completely irrelevant for another one.

Ranganathan’s idea of APUPA is founded exactly on this blurred line. He writes: “A totally or intimately relevant record may be called ‘Umbral Record’. A partially irrelevant record may be called ‘Penumbra Record’. A totally irrelevant record may be called ‘Alien Record’.” (Ranganathan, 1951, 228). Here Ranganathan establishes the equation *helpful order* is equal to *APUPA*, i.e. Alien-Penumbral-Umbral-Penumbral-Alien, and the passage between each class is determined by each user rather than being *a priori*.

**3. Main classes order in Colon Classification**

The theme of helpful order, also described as *helpful sequence* or *filiatory sequence*, returns many times in Ranganathan’s works. The order of Main Classes within his general classification scheme, Colon Classification, is not an exception, although “it is possible to have several more or less equally helpful sequences of Main Classes”, such as that used in the Vedic period, or in the Greek or the Modern periods.

Apparently, the sequence of Main Classes has no reference to APUPA. If represented by the graph of a curve, APUPA describes a bell curve, where the upper part is the full umbral class of documents. However, in describing the helpful sequence of main classes, Ranganathan uses the expression “peak” to refer to the value of the discipline of Spiritual Experience: “to highlight the peak position among the Main Classes occupied by Spiritual Experience, it is denoted in the Colon Classification by the digit ?” (Ranganathan, 1965, 40). He writes:

*the sequence among the Natural Sciences begins with the abstract discipline of Mathematics. Then follow[s] in succession the discipline of Physics dealing with matter* qua *matter and all forms of energy except chemical energy. The next is the discipline of Chemistry dealing with substances; that is, differentiated matter and chemical energy. Then follow the disciplines in which life principles are progressively intensified, in other words, the progression is from the abstract to the concrete, and from primary to integrated levels. The culmination is in the discipline of Spiritual Experience concerned with concreteness and integralness [sic] par excellence. The discipline of Spiritual Experience forms, as it were, a peak. Then follows the descent from the peak on its other side. First comes the discipline of Fine Arts, including Literature – the Fine Arts par excellence. These disciplines are soaked in Spiritual Experience and emanate from intuition. Then comes the recession into the region of greater intellection and less intuition; that is, Religion as distinct from religious experience, and Philosophy and Psychology. All these disciplines predominantly concern the individual or emanate from them. Therefore, they are collectively called Humanities. Then follow the disciplines concerned with human groups and communities. These are collectively called Social Sciences. Life in society is very complicated. The only way of living the moment in a orderly way is to regulate life by laws. The variety of legal systems in the world – almost contradicting one another at several points – suggests that the artificiality or arbitrariness and deviation from naturalness culminate in the discipline of Law. The peak of naturalness is again in Spiritual Experience. As we move down from that discipline towards the disciplines of Law, there is an increasing recession from naturalness and progression towards the arbitrariness or artificiality* (Ranganathan, 1965, 39–40).

The first half of the sequence is similar to those of other general classifications, like the Bliss Bibliographic Classification, the Russian Library-Bibliographical Classification and the Broad System of Ordering, which are ultimately inspired to the sequence of sciences outlined by 19th-century philosopher Auguste Comte, running from mathematics and physics, through chemistry, biology and psychology, to the social and human sciences. Bliss (1939) described its application to library collections "gradation in speciality", as it runs from very general disciplines to others dealing with more and more specialized parts of the world. This sequence is often associated to the sequence of integrative levels of increasing organization in nature (Mills and Broughton, 1977).

The main classes order adopted by Ranganathan for his Colon Classification is deeply original, in that it integrates this classical sequence as the first part of a larger model, again taking the shape of a bell curve. The rationale he provides for this is that of an increase in "concreteness and integralness", peaking at Spiritual Experience which takes here a central position, unusual in other classifications, then followed by "an increasing recession from naturalness and progression towards the arbitrariness or artificiality" that would be typical of human and social sciences. This second part is reversed as compared to most systems, where literature and the arts appear towards the end, as they follow rather than precede the social sciences.

To our Western culture, this approach sounds unusual, and its principle quite artificial. However, the resulting conceptual pattern is again that of a bell curve consisting of an increase, a peak and a decrease, which appears to represent a more deep and general model in Ranganathan's thought, besides the explanation he provides for main classes themselves.

A secondary feature of main classes order in Colon Classification is that most applied sciences immediately follow a corresponding pure one, rather than being grouped later under some technological classes as it happens in other systems. This choice had a precursor in 19th-century classification of sciences in the version of André-Marie Ampère (Gnoli, 2006, 139).

**4. Anteriorizing common isolates**

APUPA expresses a strong idea; it is a pattern applicable throughout the whole bibliographic universe and it is useful for arrangement at least at three different levels: a) arrangement of ideas, as it allows to obtain a useful sequence of ideas (and/or specific subjects and/or ‘expressed thoughts’ (Ranganathan 1964, 118)) relating to one disciplinary domain as well as to the whole universe of knowledge; b) arrangement of related works (relationships among different works, as among a work and its commentaries, digests, indexes, etc.); and c) arrangement of resources relating to the same intellectual or artistic content (relationships among a work and its expressions, manifestations, and items).

This approach reflects the three levels of bibliographic analysis usually required to create a catalogue that fulfil Cutter’s principles: conceptual analysis, literary analysis and formal analysis (Bianchini, 2012, 4).

The first feature is linked to arrangement of ideas, that is obtained by means of the sequence of Main Classes discussed above and by a specific facet formula for each Main Class.

The second feature is obtained by adding to the class representing a simple, compound or complex subject a special device called *common isolate*, that is defined as an isolate (i.e., an idea, a term and/or a notation) listed in special classification tables. Common isolates have the property to be attachable to many different classes (although not all common isolates can be attached to every possible class). Examples of common isolates, taken from different tables of *Chapter 2 – Common Isolate* of Colon Classification include:

a Bibliography

c Concordance

d Table

e Formula

f Investigation

f2 Observation

g Criticism

k Cyclopedia

m Periodical

n Serial

p Conference proceedings

s Statistics

v Source materials

v7 Archeology

v8 Archives

y1 Program of instruction

y2 Syllabus

y7 Case study

y8 Digest

etc.

It is not possible to examine all the rules necessary to apply common isolates to their relative class; not only because common isolates can be applied with or without a connecting symbol, before or after space facet or time facet, etc., but also because understanding common isolates requires a deep knowledge of time, space, and language isolates too (Ranganathan, 1961a, part 1, 43).

Nevertheless it must be underlined that they allow to obtain an helpful sequence of works relating to the same subject, but treating it by means of different literary forms (bibliographies, dictionaries, encyclopedias, concordances, sources, statistics, etc.). Differentiation among common isolates allows to reach the bell curve of APUPA around the interested class, because the resulting order ranges from raw and general data (e.g. dictionaries, statistics, and bibliographies) to specialization and in-depth-analysis (e.g. investigations, criticisms, etc.).

While other classification systems also include "common auxiliaries" or "common facets" to specify such kinds of formal features of a document, Ranganathan's "common isolates" again introduce some original device in classification. Indeed, the default rule in classification theory is that specification of a subject, be it formal, temporal, spatial or anything other, will always produce compound subjects to be listed *after* the subject in its simple form. Thus in e.g. Universal Decimal Classification (UDC), bibliographies of botany or tables of botany will be listed after the most general class of botany, *58*. This is taken to be an application of the basic principle that general subjects should precede specific subjects.

However, in the case of Colon Classification, common isolates are divided between *anteriorising* and *posteriorising* ones. The classification manual prescribes that anteriorizing common isolates should precede the corresponding class without the isolate: that is, a bibliography of botany should precede a general book of botany. To our knowledge, Colon is the only general classification to adopt such a rule. (Although UDC does prescribe that classes formed by summation of several disciplines, or a range of disciplines, should precede the simple one, common facets are still taken as specifications and will follow their basic class.) The rationale for this is that, in the sequence of shelved books, users should find auxiliary and introductory sources before treatments of the subject as such. The latter will thus stay at the peak of a bell curve, preceded by anteriorizing common isolates and followed by other isolates or facets.

At Ranganathan's time, the application of this prescription to printed labels or schedules was done manually. When applying it in contemporary digital forms, it implies an additional issues to the design of interfaces. Indeed, the default order in all computer systems is that a blank space precedes any other character, while Ranganathan requires that such lower-case letters as *a*, *c*, *d* ... follow it. It is then necessary to write a script that treats blank space as a character following them. This has been experimented successfully years ago by Eugenio Gatto and Fernanda Mortara during their application of Colon Classification at the Polytechnic of Turin (Gatto, pers. comm.).

**5. Book number**

In Colon Classification, shelf arrangement of resources is obtained by call numbers, that are made of a class number plus a book number. Ranganathan (1967a, 503) writes:

*A scheme for book classification should include a scheme for book numbers in order to individualize the documents having the same subject as their Ultimate Class and to mechanise their preferred arrangement among themselves”. As far as book number is unique for each resource, class numbers are useful to collocate resources with the same intellectual or artistic content and book numbers are useful to identify resources by a set of proper attributes.*

Henry E. Bliss claimed that book number is fundamental, because without it the exact designation of a book appears impossible. He noted that internal notation provides for “designating the individuals of the class, whether authors, books, or other things, and for maintaining the order of their *arrangement within* the class. It is more consistent, simpler, and more economical to regard the book as the individual to be so designated, rather than the author” (Bliss, 1939, 65).

For the first time in the history of book number, a facet formula is introduced also for book numbers in Colon Classification (Satija, 1987, 74). For book number too, each element is represented by a figure, and the reading of figures is from left to right (i.e., continuing the reading of the class number). The facet formula for book numbers is the following:

[L] [F] [Y] [A] . [V] ? [S]; [C] : [Cr]

It means that book number is composed by one or more of the following facets:

– Language number (facet [L]) ;

– Form number (facet [F]) ;

– Year number (facet [Y]) ;

– Accession Part of book number (facet [A]) ;

– Volume number (facet [V]) ;

– Supplement number (facet [S]) ;

– Copy number (facet [C]) ;

– Criticism number (facet [Cr]).

Within Colon Classification, book number denotes in fact a sequence of figures that represents a set of specific characteristics of the resource. Such characteristics have the function to individualize the single resource, that is, to allow the unique designation of its relative position among other resources that share the same ultimate class (i.e., the same class number). By the conventions adopted in Colon Classification, the representation of many relevant attributes of a resource although intellectual content—already expressed by class number—becomes possible (Bianchini, 2012, 8).

Also after such a brief examination of the facet formula for the book number, it is clear that the arrangement of documents on the shelves is based on the correspondence of the couple of facets *Language* + *Form* with the two main attributes of entity Expression. In the same way, the third facet is *Year*, and it provides for a major attribute of the entity Manifestation (Bianchini, 2010, 30–31).

Application of facet formula for book number to the many expressions and manifestations of a work results in an arrangement of FRBR entities out of reach to present OPACs – in depth discussed and exemplified in – but directly available to users on the shelves.

**6. Bell-shaped models in other disciplines**

The scope and objectives of shelf arrangement are enlightened in *Reference Service*, where Ranganathan describes the work of the reference librarian and his correct approach towards the user in explaining the arrangement of resources about a specific subject (or *ultimate class*) on the shelves:

*Without in any way embarrassing him by the use of the classifier’s jargon, it should be impressed upon freshman’s mind that apart from the books, placed in the ultimate class in question, information on his specific subject could also be found in parts or chapters of books falling within classes of greater extension. He can be helped to realise that they precede his ultimate class. He should further be told that information on some particular aspects of his specific subject could also be found in the books in classes of smaller extension than in his ultimate class. Taking larger strides on either side of the specific subject, forming his centre of interest, it must be possible demonstrate to the freshman the helpful sequence in which the main subjects themselves fall in the scheme of classification in use. Speaking in general terms, the freshman can thus be made to sense and appreciate the APUPA pattern (Alien-Penumbral-Umbral-Penumbral-Alien pattern) provided be the scheme of classification* (Ranganathan, 1961b, 93).

Ranganathan offers here a really nice representation of a user's experience in front of a shelf. The progression from penumbral to umbral seems counterintuitive as compared to Western habit to represent cognitive processes as progressions from shadow to light (cfr. the idea of *enlightening*). We also notice the permanence of the Latin term *umbra* instead of English *shadow*.

With its concentric distribution of bands, *umbral*, *penumbral* and *alien*, the bell pattern can suggest analogies with other bell models in two disciplinary fields:

– Cognitive science: the bell of human field of vision, with similar concentric bands *frontal* to *peripheral*, up to an area beyond the limits of the visual field which is delegated to another sense: hearing. We are led to this «visual interpretation» of the words *umbral* and *penumbral* also by Ranganathan’s frequent use of visual analogies in presenting classification principles – e.g. his choice of the term *focus* (Ranganathan, 1951, sect. 147) or *Beacon light* vs *Search-Light* analogy (Ranganathan 1951, sect. 1722).

– Mathematics: first of all as for the mentioned Gaussian bell distribution model; but also as for the branch of mathematics known as umbral calculus, and sources close to the mathematics studied by Ranganathan in his Madras years.

We try to follow the two approaches above, to see if these suggestions can have any relationship with the genesis of the APUPA pattern.

***6.1 Cognitive science: modularity, information overload and peripheral vision***

The APUPA formula does not appear in any work of Ranganathan before the publication of *Classification and communication* (Ranganathan, 1951). Perhaps this concept was not part of his early process of reinvention of classification in the 1930s. It was probably defined later, in conjunction with the subsequent innovations related to the scientific debate and the events of the time.

This refers in particular to the great acceleration that World War II gave to research on information science and its applications to telecommunication; in particular to the issues of telecommunication, message encryption and information retrieval, from Turing to Bush (1945), Shannon & Weaver (1949) and Wiener (1950). All these technologies had a military origin, and would have been later applied to civil information science and discussed in a proliferation of international organizations and meetings about and national documentation centres[[2]](#footnote-1), and in a large number of sociological studies, up to McLuhan (1951).

Ranganathan probably could learn about this research because it was often invited in the UK and the US in the years immediately following II World War and immediately preceding to the drafting of *Classification and Communication*. In fact in “*1948 he toured the United Kingdom at the invitation of the British Council and lectured at many library schools, and he triggered the spirit of some kindred souls to found the Classification Research Group (London)"* (Drake, 2005, 2422). He was invited in the USA by UNESCO, United Nations and foundations, and about this experience he wrote:

*It was early 1950. The Rockefeller Foundation extended an invitation to visit USA. It was accepted chiefly to observe documentation in action at close quarters in several industries in the USA. But it was learnt that the purpose of the Foundation inviting me was to explore the role of classification in communication*” (Ranganathan, 1963, 18).

That was the occasion for creating and writing *Classification and communication* (Gopinath, 1994, x).

In particular, Shannon and Weaver’s mathematical model of communication[[3]](#footnote-2) appears to be similar to the model used by Ranganathan in the analysis of a whole range of media, that he develops in the first chapter of the third part of *Classification and communication* (Ranganathan, 1951, 22-27; 31). Here he especially reflects on container *modularity*, a characteristic of media like alphabet and books, which interposes between the message and the recipient: on the one hand, modularity guarantees the best transmissibility and storability and therefore enhances the capacity of the channel; on the other hand, modularity complicates their perceivability and therefore their selection, closing them in containers that require a decoding activity.

The combination of these two features exposes to the risk of jamming, due to overload. All that needs to be put into relation with the increasing quantities, specialization and diversity of information flows that followed the acceleration triggered by the World Wars, with retrieval of information from large numbers of documents, in particular “*micro subjects* *embodied in articles in periodicals or scientific reports*” (Ranganathan, 1967a, sect. XG14). It is also necessary to consider that at that time a library must not have displayed the variety of graphic and editorial formats it offers today, but sequences of far more homogeneous objects.

In the classification process, Ranganathan sees an opportunity to face these issues: he analyzes classification as if it were a *medium*, using the above-mentioned mathematical model of communication. It is at this point that Ranganathan introduces APUPA as a fundamental element of this medium:

*Classification is thus a transformation of thought. This change in arrangement from accession or random order to APUPA order is itself a subsidiary transformation. It is the first use of classification in the process of communication. […] The transformer is the classifier. The means is the Classification Scheme.* (Ranganathan, 1951, sect. 3212)

The title of section 325, *Classification will have a future as a means of communication*,summarizes the central vision of the book: the strength of classification lies in its ability to counter the noise of the overload of stimuli and mostly of those which are irrelevant to the searcher. At the heart of this feature is the APUPA pattern, which seems to be able to bring the experience of accessing information through books to a kind of organization more compliant with our perceptual system; it manages to do this through a sequential arrangement, which allows humans to act their own signal selection strategies.

So Ranganathan seems to strengthen the model of Shannon and Weaver in the direction of message encoding, which a little later (Schramm 1954) would characterize the evolution of the communication models in a semiotic sense.

To summarize, the translation of the predominant subject of a text as stated in very few words into a sortable number is a complex operation; the media of the time, as well as today's computers, are not able to perform this task. As far as information overload is getting stronger, Ranganathan’s vision is more and more important for us (Carr, 2010, chap. 7).

Ranganathan’s reflection often reaches the most subtle and microscopic energy levels, “activated by librarians who can transform the potential energy stored in books into kinetic energy in the minds of readers” (Ranganathan, 1946, sect. 116):

*inherent tendency to arrange is a concomitant of the finiteness of the speed of nervous impulses in the human body. When speed is finite, structure is inevitable* (Ranganathan, 1967a, sect. XB1).

To sequence and prioritize what is necessary, a «knowledge economy» is needed, which involves being very acute where appropriate, but also knowing how to simplify[[4]](#footnote-3) in order not to overreach the limits of stimuli processing capabilities, which would lead to information overflow and passiveness.

The correspondence between visual perception and APUPA seems to occur at a deep level, rather than on the surface; another possible analogy is that with the filtering strategy in vision that has evolved two different types of receptors (cones and rods) operating in different bands, respectively frontal and peripheral[[5]](#footnote-4).

As in APUPA, umbral and penumbral affirm a strict knowledge economy that differentiates whether and how information is collected in different areas; indeed, having too many details on everything is not compatible with the limits of human processing capabilities.

During World War II great acceleration, many scientists sensed that apparently exceeded limits persisted in inner limits on the signal processing level. Providing some sorting by relevance states a principle of proportion between stimuli and senses, which enables people to use their most basic selection strategies. This is what Ranganathan seems to be saying where he compares user experience in closed shelf libraries with that in some commercial shops (Ranganathan, 1931, sect. 513):

the former assumes a single use case, in which users know exactly what they are looking for;

the latter aims to user engagement,knowing that only rarely is what is desired by users immediately clear and expressible in words; more often the so-called demand is defined by trial while observing an available offer (cfr. the woman looking for a new sari in Ranganathan 1967a, sect. XA6). This can be implemented by leaving users free to lose their way and, at the same time, helping them to discover autonomously by providing continuous feedback.

Peripheral vision plays a fundamental role in this research[[6]](#footnote-5). Often, the stages of perception and selection are not distinct but are interrelated, inseparable phases. This (rather than a discovery "by chance") generates the feeling of unexpected coincidence called *serendipity*. Perhaps Ranganathan's tendency to make analogies that hold together the levels of visual perception and of knowledge organization is an expression of that inner connection between levels. As Vannevar Bush pointed out in the same years (Bush, 1945), our mind’s associations can rarely be traced back to a logical tree; more often they proceed by analogical leaps.

***6.2 Mathematics: umbral and differential calculus***

Searching for the texts in which Ranganathan talks about Gauss, we could not find any reference to "normal distribution". At the moment, it does not appear possible to push the analogy with the Gaussian curve beyond a suggestion to represent the pattern with a bell-shaped graph.

The term *umbral* could also have been derived from the work on umbral calculus developed in Madras by Indian mathematician Srinivasa Ramanujan. The label *umbral calculus* was introduced in the second half of 19th century by English mathematician James Joseph Sylvester:

*The term ‘umbral calculus’ was coined by Sylvester from the word ‘umbra’ (meaning ‘shadow’ in Latin), and reflects the fact that for many types of identities involving sequences of polynomials with powers , ‘shadow’ identities are obtained when the polynomials are changed to discrete values and the exponent in is changed to the falling factorial* (Weisstein, s.d.)*.*

In a 1903 essay, Charles Sanders Peirce criticizes the decision to base the analogy on the shadow, arguing that would better analogies based on images of the radicals and ions:

*Sylvester's name umbra, which is the only distinctive name the thing has ever received, must, I fear, be retained, although ion or radicle would be far better. For who ever heard of two shadow combining together to form a substance! (…) In other mathematics, they have no existence in the universe of quantity. But joined together in sets they do. They are just like chemicals radicals, each having a certain number of unsatisfied wants. When each of these is satisfied by union with another, the completely saturated whole has an existence in the universe of quantity.*

Few words before, the father of pragmatism and founder of modern semiotics specifies:

*what Sylvester called ‘my umbral notation’ had first been published in 1693 by another man of some talent, named Godfry William Leibniz.* (Peirce, 1997, 125).

Leibniz was a rare example of total genius (and librarian) with the capability of translating into mathematical form his philosophical point of view on human intellect. As explained in a recent study:

*To understand what this theory of the differential relation n terms, consider the corresponding theory of perception that Leibniz develops in relation to it. Leibniz had observed that we often perceive things of which we are not consciously aware. We recall a familiar scene and become aware of a detail we did not notice at the time; the background noise of a dripping faucet suddenly enters our consciousness at night. Leibniz therefore draw a distinction between conscious perceptions («apperceptions», or molar perceptions) and unconscious perceptions («minute» or molecular perceptions) (…) a conscious perception is produced when at least two of these minute and « virtual » perceptions enter into a differential relation that determines a singularity: that is, a conscious perception. (…) the calculus thus function in Leibniz as the physic mechanism of perception* (Smith, 2012, 54).

Leibniz’s work on differential calculus (1684) resulted very obscure to mathematicians. The Bernoulli brothers were the first to try to understand and apply Leibniz’s theories and became two of his main correspondents (Swetz et al., 2011). Bernoulli numbers were an important step in the history of umbral calculus, and were investigated and developed by Srinivasa Ramanujan; he made important discoveries on this branch of algebra, like *mock modular forms,* with the help of G.H. Hardy in Cambridge and Edward B. Ross in Madras[[7]](#footnote-6). The latter was professor of mathematics of Ranganathan at University and one the most important person in his life: indeed, Ranganathan credited Ross as the real discoverer of the Fifth Law of library science and created a scholarship at the Madras Christian College (Ranganathan, 1967a, sect. DB06; Bianchini, 2015, 30 and 36-37). Ranganathan traveled to England in March 1925 to meet Hardy and receive Ramanujan’s handwritten notebooks (Berndt, 1985, 5) to be retained in the University of MadrasLibrary. Ranganathan devoted a biography to Ramanujam (Ranganathan, 1967*b*) and tried to evoke him in séances (Kumar, 1992, 46).

***6.3 A synthesis between mathematics and cognitive science?***

While the progression from penumbral to umbral could be perceived as counterintuitive, it is not so in a vision that combines mathematics and cognitive science, where the challenge is to integrate the conscious and the unconscious:

*Descartes’s principle of «clear and distinct» ideas is broken down into two irreducible values, which can never be reunited to constitute a «natural light»: conscious perceptions are clear but confused (not distinct), while unconscious perceptions (ideas) are distinct but necessarily obscure (not clear). Indeed Leibniz can be said to have developed one of the first theories of the unconscious* (Smith, 2012, 55).

Convergence of usually distant levels such as the phenomenal and the abstracted, the pragmatic and the theoretical, the extremely small and the extremely large, that characterize Leibniz (Smith, 2012, 55), seems to be a common attitude in Ranganathan, including:

his choice of the colon as a key symbol in his new classification scheme:

*The semantically rich digits used in the Colon Classification are normally a to z, 1 to 9, and A to Z. Thus the digit used to separate one facet number from the succeeding facet number should have an ordinal value less than that of one. At this juncture the fact that zero is elliptical in shape gave a clue. According to the Geometry of Conics a point-pair may be deemed to be a degenerated form of an ellipse. This suggested that a point-pair may be taken to be a “greater zero”; in other words, as a digit with an ordinal value between those of zero and one. Sayers agreed with the idea of using (:) colon as the digit needed for insertion between any two facet numbers. (…) It was called “Colon Classification” to emphasize the rich potential added to the scheme by the semantically poor digit (:) colon* (Ranganathan, 1965, 14-15)

his awareness of the necessity to reconcile the potentially infinite expansion of possible approaches to the issue (*foci*) with the human limits in development and fruition of classification systems (Ranganathan, 1967a, sect. RA);

the sensitivity with which he observes the user’s perceptual experience in front of the shelf, leading to the necessity of open shelves and to care about every detail that will encourage the widest variety of approaches to the experience (Ranganathan, 1931, sect. 513; 1967a, sect. XA6).

**7. Conclusions**

The APUPA pattern goes far beyond the moment of the “face to face” between man and books in front of a single shelf, an interaction that is felt by Ranganathan to be fragile and in need of help (“The affinity between books and men is weak. It needs the catalytic action of library staff to get activated”, Ranganathan, 1960, sect. 2862). Indeed, the application of APUPA can be extended to larger contexts, such as the spatial arrangement of shelves in the room and the corresponding main classes. In fact, when describing the APUPA pattern for the first time, Ranganathan (1951, sect. 321) uses the term *fanned out*, that refers to a radial distribution all around the space rather than a simple linear sequence*.*

Moreover, we have seen in Section 3 that APUPA applies not only at the level of "ultimate classes", but also at that of main classes. Indeed, the organization of main classes into two groups enables perception and memory to work with two sequences of 12 classes instead of one sequence of 24. The two sequences are also placed on distinct semantic axes (respectively, increasing phenomenal complexity and increasing artificiality) converging towards a sort of summit, which is emphasized by the uppercase delta symbol representing the class of Spiritual Experience and Mysticism, the apex of complexity and naturalness. In such choices, Ranganathan combines spirituality with pragmatism. Strong references to helping users can also be found in the awareness of the limits of human memory, which will be theorized later in the notion of *futility point* (Blair, 1980). This approach leads the user to be aware of the position of the book within the whole of knowledge, putting it into a knowledge map that is also an invitation to explore contiguity or jumps in subjects.

The APUPA scheme thus seems to act on multiple depth levels, from macro to micro: from the arrangement of mainclasses in the room (Section 3of this article) to book number (Section 5), through the arrangement within the same ultimate class (Section 4). In all cases, it defines a clear logic of distribution that can be easily perceived by peripheral vision, and encourages the identification of landmarks, thus making the subsequent focusing work simpler.

The fact that APUPA can act at various levels should ensure good scalability in the concrete application of the pattern, making it effective in both large libraries with many documents and small libraries with documents focused on a limited number of classes. The perception of this texture of meanings contributes to the *serendipity* that makes the experience of browsing shelves enjoyable.

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1. Emphasis is added by the authors. [↑](#footnote-ref-0)
2. “*During World War 11, the importance of the dissemination of the latest scientific information to research workers was realised. As a result of this, national documentation centres were organised in many newly developing countries. A filip to this movement was also given by the initiative taken by the Unesco. One of the objectives of the Unesco was to assist in the elimination of barriers in international communication. Recognising that "wars are made in the minds of men", Unesco initiated a programme of developing public libraries and documentation centres as a means of ‘improving the minds of men’*." (Ranganathan, 1963, sect. C3) [↑](#footnote-ref-1)
3. «In 1964 the University of Pittsburgh conferred a Doctor of Letters to Ranganathan on him along with Shannon and Weaver» (*Encyclopedia of Library History*, 338, *sub voce* "Library and Information Science research"). [↑](#footnote-ref-2)
4. Barfield et. al. (1990) argue that excessive opening of the field of vision (FOV), which sometimes takes place in simulators or video games, could generate an opposing problem of information overload: "participants judge azimuth and elevation under different conditions of field of view. The authors reported that performance was best under midsized FOV conditions (45 or 60 degrees) and worse under extreme FOV conditions (30 or 75 degrees)". [↑](#footnote-ref-3)
5. (1) front-end view, very narrow and very rich in cones, is capable of building the image with higher resolution and enriched with colour, to support the most challenging interactions with what we focus precisely before the eyes; (2) all around it an area of peripheral vision is developed, in which cones are replaced by rods, more sensitive in low-light conditions and in the perception of movement, but with a poor definition of the image and without color. (3)More externally, the gradually disappear receptors follow a bell curve: the vision is replaced by the sense of hearing, which has a radically alien structure to that of the sense of sight; regardless of day and night, only warning in very significant cases. (Cfr. an introduction available at http://hyperphysics.phy-astr.gsu.edu/hbase/vision/rodcone.html#c2; a more detailed text is Goldstein (2009, 50 and following). [↑](#footnote-ref-4)
6. Czerwinski et al. (2002), studying the human ability to consolidate experience in the context of maps, suggest that this may be inhibited by limitation of peripheral vision, in particular by absence of relationship between specific experience and the context of a wider network of reference points (landmarks)*.* Dolezal (1982) describes the effects of restricting view field to 12 degrees, including disorientation, dizziness during rapid head movements, difficulty in tracking objects, and difficulty in forming a cognitive map of unfamiliar places. He observes that eye-hand coordination is impaired, and ability to integrate visual information across successive views is greatly reduced. [↑](#footnote-ref-5)
7. Ross had studied at Cambridge with G.H. Hardy, who was soon to become one of the greatest admirers of Ramanujan*.* “*In 1913, the English mathematician G. H. Hardy received a letter from an accounting clerk in Madras, India, describing some mathematical formulas he had discovered. Many of them were old hat, and some were flat-out wrong, but on the final page were three formulas that blew Hardy’s mind. “They must be true,” wrote Hardy, who promptly invited the clerk, Srinivasa Ramanujan, to England, “because, if they were not true, no one would have the imagination to invent them. (…)* *As he lay dying in India at age 32, he wrote Hardy another letter saying that he had discovered what he called “mock theta” functions, which entered into mathematics ‘beautifully’. (…) A mock modular form always has an associated modular function called its “shadow,” so they named their hypothesis the Umbral Moonshine Conjecture—umbra is Latin for ‘shadow.’*”(Klarreich, 2015). [↑](#footnote-ref-6)