Kinetic models of opinion formation

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Porto Ercole, June 8-10 2008
Summer School METHODS AND MODELS OF KINETIC THEORY
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Collective phenomena


The collective behaviors of a society composed by a sufficiently large number of individuals (agents) described using the laws of statistical mechanics as it happens in a physical system composed of many interacting particles.

The details of the social interactions between agents then characterize the emerging statistical phenomena.
Opinion formation


- The starting point of a large part of these models, however, is represented by a cellular automata, where the lattice points are the agents.

- Any of the agents of a community is initially associated with a random distribution of numbers, one of which is the opinion.

- Society is modelled as a graph, where each agent interacts with his neighborhoods in iterative way.
Compromise process

- Other attempts have been successfully applied [Ben-Naim E. (2005), Slanina F. Lavička H. (2003)].
- Describe formation of opinion by means of mean fields model equations.
- In general partial differential equations of diffusive type, treated analytically to give explicit steady states.
- [Ben-Naim E. (2005)] focused on two aspects of opinion formation, which could be responsible of the formation of coherent structures.
The diffusion process, which allows individual agents to change their opinions in a random diffusive fashion.

The compromise process has is basis on the human tendency to settle conflicts.

Diffusion accounts for the possibility that people may change opinion through a global access to information.

Aspect gaining in importance due to the emerging of new possibilities (among them electronic mail and web navigation [Rash W. Politics on the nets: wiring the political process 1997]).

We consider here a class of kinetic models of opinion formation, bases on two-body interactions involving both compromise and diffusion properties in exchanges between individuals.
The kinetic model gives in a suitable asymptotic limit (hereafter called quasi-invariant opinion limit) a partial differential equation of Fokker-Planck type for the distribution of opinion among individuals.


The equilibrium state of the Fokker-Planck equation can be computed explicitly and reveals formation of picks in correspondence to the points where diffusion is missing.

The mathematical methods close to kinetic theory of granular gases, where the limit procedure is known as quasi-elastic asymptotics [McNamara S., Young W.R. (1993), Toscani G. (2000)].
Related results

- Similar analysis on a kinetic model of a simple market economy with a constant growth mechanism [Cordier S., Pareschi L., Toscani G. (2005), Slanina F. (2004)].

- Formation of steady states with Pareto tails [Pareto V. *Cours d’Economie Politique* 1897].

The goal of the forthcoming kinetic model of opinion formation, is to describe the evolution of the distribution of opinions in a society by means of microscopic interactions among agents or individuals which exchange information.

We associate opinion with a variable $w$ which varies continuously from $-1$ to $1$, where $-1$ and $1$ denote the two (extreme) opposite opinions. We moreover assume that interactions do not destroy the bounds, which corresponds to impose that the extreme opinions cannot be crossed.

Denote by $(w, w^*)$, with $w, w^* \in \mathcal{I}$ the pair of opinions of two individuals before the interaction and $(w', w'^*)$ their opinions after exchanging information between them and with the exterior.
Let $I = [-1, +1]$ denote the interval of possible opinions. We describe the binary interaction by the rules

$$w' = w - \gamma P(|w|)(w - w^*) + \eta D(|w|),$$
$$w'^* = w^* - \gamma P(|w^*|)(w^* - w) + \eta^* D(|w^*|).$$

- Opinions **cannot** cross boundaries. The interaction takes place only if both $w', w^* \in I$.
- $\gamma \in (0, 1/2)$ is a given constant. $\eta$ and $\eta^*$ random variables with the same distribution with variance $\sigma^2$ and zero mean, taking values on a set $B \subseteq \mathbb{R}$.
- The constant $\gamma$ and the variance $\sigma^2$ measure respectively the compromise propensity and the modification of opinion due to diffusion.
- The functions $P(\cdot)$ and $D(\cdot)$ describe the local relevance of the compromise and diffusion for a given opinion.
Details of binary exchange

- The first part is related to the **compromise propensity** of the agents.
- The second contains the **diffusion effects of external events**.
- The pre-interaction opinion $w$ increases (getting closer to $w_*$) when $w_* > w$ and decreases in the opposite situation.
- The presence of both the functions $P(\cdot)$ and $D(\cdot)$ is linked to the hypothesis that the availability to the change of opinion is linked to the opinion itself, and decreases as soon as one get closer to extremal opinions.
- Extremal opinions are **more difficult to change**.
- Assume $P(|w|)$ and $D(|w|)$ non increasing with respect to $|w|$, and in addition $0 \leq P(|w|) \leq 1$, $0 \leq D(|w|) \leq 1$. 
Particular cases

- In absence of the diffusion contribution ($\eta, \eta_* \equiv 0$),

$$w' + w'_* = w + w_* + \gamma(w - w_*)(P(|w|) - P(|w_*|))$$
$$w' - w'_* = (1 - 2\gamma(P(|w|) + P(|w_*|)))(w - w_*).$$

- If $P(\cdot)$ is not constant, $P = 1$, the total momentum is not conserved and it can increase or decrease depending on the opinions before the interaction.

- If $P(\cdot)$ is assumed constant, the interaction correspond to a granular gas like interaction (or to a traffic flow model [Klar A., Wegener R. (1996)])

- In a single interaction, the compromise propensity implies that the difference of opinion is diminishing, with $|w' - w'_*| = (1 - 2\gamma)|w - w_*|$. Thus all agents will end up in the society with exactly the same opinion.
Particular cases II

- In this elementary case a **constant part of the relative opinion is restituted** after the interaction.
- In the other cases

$$|w' - w'| = (1 - 2\gamma(P(|w|) + P(|w_*|))) |w - w_*|.$$ 

- This implies

$$0 \leq \varepsilon(w, w_*) = 1 - 2\gamma(P(|w|) + P(|w_*|)) \leq 1.$$ 

- The general case corresponds to a granular gas interaction with a variable coefficient of restitution [Toscani G. (2000)].
- In absence of diffusion, the **lateral bounds are not violated**

$$w' = (1 - \gamma P(|w|))w + \gamma P(|w|)w_*$$
Let $f(w, t)$ denote the distribution of opinion $w \in I$ at time $t \geq 0$.

Standard methods of kinetic theory [Cercignani et al. *The mathematical theory of dilute gases* 1994] allow to describe the time evolution of $f$ as a balance between bilinear gain and loss of opinion terms

$$\frac{\partial f}{\partial t} = \int_{\mathcal{B}^2} \int_{\mathcal{I}} \left( \beta \frac{1}{J} f'(w)f'(w_*) - f(w)f(w_*) \right) dw_* d\eta d\eta_*$$

$('w', w_*)$ are the pre-interaction opinions that generate the couple $(w, w_*)$ of opinions after the interaction.

The kernels $'\beta$ and $\beta$ are related to the details of the binary interaction.
The transition rate is taken of the form

\[ \beta_{(w, w)} \rightarrow (w', w') = \Theta(\eta)\Theta(\eta_*)\chi(|w'| \leq 1)\chi(|w'_*| \leq 1), \]

- \( \chi(A) \) is the indicator function of the set \( A \), and \( \Theta(\cdot) \) is a symmetrical probability density with zero mean and variance \( \sigma^2 \).
- The rate function \( \beta_{(w, w)} \rightarrow (w', w') \) characterizes the effects of external events on opinion through the distribution of the random variables \( \Theta \) and \( \Theta_* \).
- The support \( B \) of the symmetric random variable is a subset of \( I \), to prevent diffusion to generate a complete change of opinion.
Simplifications

- The main problem in opinion dynamics relies in looking for the formation of stationary profiles for the opinion.
- In the kinetic picture this corresponds to look for the large time behavior of the density of opinion $f(w,t)$.
- Extremely difficult problem to describe in details the large-time behavior of the solution for a general kernel.
- Restrict the analysis to the cases in which the kernel $\beta$ does not depend on the opinion variables (Maxwellian case).
- Any choice of $D(|w|)$ and $B$ which preserve the lateral bounds of extreme opinions allows to study in details the dynamics of the model with a significant simplification.
Simplifications II

- The Maxwellian assumption implies

\[ \beta(w, w_*) \rightarrow (w', w'_*) = \beta(\eta, \eta_*) = \Theta(\eta)\Theta(\eta_*) . \]

- In this case the weak form reads

\[ \frac{d}{dt} \int_{I} \phi(w)f(w, t) \, dw = \left\langle \int_{I^2} f(w)f(w_*)(\phi(w') - \phi(w)) \, dw_* \, dw \right\rangle \]

- Conservation of the total opinion is obtained for \( \phi(w) = 1 \), which represents in general the only conservation property satisfied by the system.

- The choice \( \phi(w) = w \) is of particular interest since it gives the time evolution of the average opinion.
We have

$$\frac{d}{dt} \int_{\mathcal{I}} w f(w, t) \, dw = \left\langle \int_{\mathcal{I}^2} f(w) f(w^*) \gamma(P(|w|)w^* - P(|w|)w) \, dw^* \, dw \right\rangle$$

$$+ \left\langle \int_{\mathcal{I}^2} f(w) f(w^*) \eta D(|w|) \, dw^* \, dw \right\rangle$$

In case $P(|w|) = 1$, the first contribution disappears. By symmetry

$$\left\langle \int_{\mathcal{I}^2} f(w) f(w^*) \gamma(w^* - w) \, dw^* \, dw \right\rangle = 0.$$
Evolution of average opinion

- Since $\langle \eta \rangle = 0$ implies

$$\frac{d}{dt} \int_I wf(w, t) \, dw =$$

$$= \langle \eta \rangle \int_{I^2} \chi(|w'| \leq 1) \chi(|w_*'| \leq 1)D(|w|)f(w)f(w_*)dw_* \, dw = 0.$$

- $P$ constant implies conservation of the average opinion.
- In general

$$\frac{d}{dt} \int_I wf(w, t) \, dw$$

$$= \gamma \int_I P(|w|)f(w) \, dw \int_I wf(w) \, dw - \gamma \int_I wP(|w|)f(w) \, dw.$$

- The evolution of the average opinion is not closed.
Fix now $\phi(w) = w^2$.

\[
\frac{d}{dt} \int_I w^2 f(w, t) \, dw = \gamma^2 \int_{I^2} P(|w|)^2 (w - w_*)^2 f(w)f(w_*) \, dwdw_*
\]

\[-2\gamma \int_{I^2} P(|w|)w(w - w_*)f(w)f(w_*) \, dwdw_* + \sigma^2 \int_I D(|w|)^2 f(w)dw.\]

Choosing $P(|w|) = 1$, $m$ the constant value of the average opinion

\[
\frac{d}{dt} \int_I w^2 f(w, t) \, dw = -2\gamma(1 - \gamma) \left[ \int_I w^2 f(w) \, dw - m^2 \right] + \sigma^2 \int_I D(|w|)^2 f(w)dw.
\]
Towards a simpler model

- Previous analysis shows that in general it is quite difficult both to study in details the evolution of the opinion density, and to describe its asymptotic behavior.

- For a general kernel the mean opinion is varying in time.

- Asymptotics of the equation result in simplified models (generally of Fokker-Planck type)

  Particularly relevant in case of a good approximation of the stationary profiles of the kinetic equation.

- Physical basis to these asymptotics comes from the interaction rule.

  Assume $P(|w|) = 1$, so that conservation both of mass and momentum holds.
Simpler models II

- The interaction rule has following properties

\[
\langle w' + w'_* \rangle = w + w_*, \quad \langle w' - w'_* \rangle = (1 - 2\gamma)(w - w_*) .
\]

- The first equality is the **property of mean conservation of opinion**.
- The second refers to the compromise propensity, which plays in favor of the **decrease (in mean) of the distance of opinions after the interaction**.
- Same property in a collision between molecules in a granular gas. There \( e = 2\gamma \) is called **coefficient of restitution** [Toscani G. (2000)].
- Suppose most of the interactions produce a **very small exchange of opinion** (\( \gamma \to 0 \)), while mean properties remain true at a macroscopic level.
Simpler models II

- While $\gamma \to 0$

\[
\left\langle \int_{I^2} (w' + w'_*) f(w)f(w_*) \, dw \, dw_* \right\rangle = 2 \int_I w f(w) \, dw = 2m(t)
\]

remains constant,
- and

\[
\frac{1}{2} \left\langle \int_{I^2} (w' - w'_*)^2 f(w)f(w_*) \, dw \, dw_* \right\rangle = \int_I w^2 f(w) \, dw - m_0^2 = C_f(t)
\]

varies with time.
- Then $m(t) = m_0$ independently of the value of $\gamma$. Moreover

\[
\frac{dC_f(t)}{dt} = -2\gamma(1 - \gamma) C_f(t) + \sigma^2 \int_I D(|w|)^2 f(w) \, dw.
\]
Scaling

1. Set

\[ \tau = \gamma t, \quad g(w, \tau) = f(w, t). \]

2. Then \( f_0(w) = g_0(w) \), and

\[ \frac{dC_g(\tau)}{d\tau} = -2 \left(1 - \gamma \right) C_g(\tau) + \frac{\sigma^2}{\gamma} \int_I D(|w|)^2 f(w) dw. \]

3. Let \( \gamma \to 0 \) and \( \sigma \to 0 \) but \( \sigma^2/\gamma = \lambda \),

\[ \frac{dC_g(\tau)}{d\tau} = -2C_g(\tau) + \lambda \int_I D(|w|)^2 f(w) dw. \]

4. \( t = \tau/\gamma \), and \( \gamma \to 0 \) describes the large-time behavior of \( f(v, t) \).
Since \( f(w, t) = g(w, \tau) \) the large-time behavior of \( f(w, t) \) is close to the large-time behavior of \( g(w, \tau) \).
Mathematical assumptions

Let

\[ M_p(A) = \left\{ \Theta \in M_0 : \int_A |w|^p d\Theta(w) < +\infty, \ p \geq 0 \right\} \]

the space of all Borel probability measures of finite momentum of order \( p \), equipped with the topology of the weak convergence of the measures.

Let \( F_s(I) \), be the class of all real functions \( h \) on \( I \) such that \( h(\pm 1) = h'(\pm 1) = 0 \), and \( h^{(m)}(v) \) is Hölder continuous of order \( \delta \),

\[ \| h^{(m)} \|_{\delta} = \sup_{v \neq w} \frac{|h^{(m)}(v) - h^{(m)}(w)|}{|v - w|^{\delta}} < \infty, \]

the integer \( m \) and the number \( 0 < \delta \leq 1 \) are such that \( m + \delta = s \), and \( h^{(m)} \) denotes the \( m \)-th derivative of \( h \).
The scaled density $g(v, \tau) = f(v, t)$ satisfies the equation

$$\frac{d}{d\tau} \int_I g(w) \phi(w) \, dw = \frac{1}{\gamma} \left\langle \int_{I^2} g(w) g(w_*)(\phi(w') - \phi(w)) \, dw_* \, dw \right\rangle.$$

- Set $\phi \in \mathcal{F}_{2+\delta}(I)$.
- By the interaction rule

$$w' - w = \gamma(w_* - w) + \eta D(|w|) \ll 1.$$

- Use a second order Taylor expansion of $\phi$ around $w$

$$\phi(w') - \phi(w) = (\gamma(w_* - w) + \eta D(|w|)) \phi'(w) + \frac{1}{2} (\gamma(w_* - w) + \eta D(|w|))^2 \phi''(\tilde{w}).$$
Inserting this expansion in the collision operator

\[
\frac{d}{d\tau} \int_I g(w)\phi(w) \, dw = \left\langle \frac{1}{\gamma} \int_I \left[ (\gamma(w_* - w) + \eta D(|w|))\phi'(w) \right. \right.
\]
\[+ \frac{1}{2} \left( \gamma(w_* - w) + \eta D(|w|) \right)^2 \phi''(w) \left. \right] g(w)g(w_*) \, dw_* \, dw \right\rangle + R(\gamma, \sigma)
\]

\( R(\gamma, \sigma) = \frac{1}{2\gamma} \left\langle \int_I \left( \gamma(w_* - w) + \eta D(|w|) \right)^2 \cdot \left( \phi''(\tilde{w}) - \phi''(w) \right) g(w)g(w_*) \, dw_* \, dw \right\rangle. \)
One shows that $R(\gamma, \sigma)$ converges to zero as as both $\gamma$ and $\sigma$ converge to zero, in such a way that $\sigma^2 = \lambda \gamma$.

Within the same scaling

$$\lim_{\gamma \to 0} \frac{1}{\gamma} \left< \int_{\mathcal{I}^2} [\gamma(w_\star - w) + \eta D(|w|))\phi'(w) + \right.$$

$$\left. + \frac{1}{2} (\gamma(w_\star - w) + \eta D(|w|))\phi''(w)]g(w)g(w_\star)dw_\star dw \right>$$

$$= \int_{\mathcal{I}} \left[ (m - w)\phi'(w) + \frac{\lambda}{2} D(|w|)\phi''(w) \right] g(w)dw$$

Since $\phi \in \mathcal{F}_s(\mathcal{I})$, weak form of the Fokker-Planck equation

$$\frac{\partial g}{\partial \tau} = \frac{\lambda}{2} \frac{\partial^2}{\partial w^2} (D(|w|)^2g) + \frac{\partial}{\partial w} ((w - m)g).$$
Main result

- We proved

**Theorem**

*Let the probability density* \( f_0 \in \mathcal{M}_0(\mathcal{I}) \), *and let the symmetric random variable* \( Y \) *which characterizes the kernel have a density in* \( \mathcal{M}_{2+\alpha} \), *with* \( \alpha > \delta \). *Then, as* \( \gamma \to 0, \sigma \to 0 \) *in such a way that* \( \sigma^2 = \lambda \gamma \) *the weak solution to the Boltzmann equation for the scaled density* \( g_\gamma(v, \tau) = f(v, t) \), *with* \( \tau = \gamma t \) *converges, up to extraction of a subsequence, to a probability density* \( g(w, \tau) \). *This density is a weak solution of the Fokker-Planck equation, and it is such that the average opinion is conserved.*
Other Fokker-Planck models

- Previous Theorem can be extended to interaction rules with a general function $P(|w|)$.
- The main difference is the evaluation of the first order term

$$\int_{\mathcal{I}^2} P(|w|)(w_* - w)\phi'(w)g(w)g(w_*)dw_* dw$$

- Let $m(\tau)$ be the value of the average opinion at time $\tau \geq 0$

$$m(\tau) = \int_{\mathcal{I}} wg(w, \tau)dw.$$  

- The scaling $t \rightarrow \tau$ implies

$$\frac{dm(\tau)}{d\tau} = m(\tau) \int_{\mathcal{I}} P(|w|)g(w, \tau) dw - \int_{\mathcal{I}} wP(|w|)g(w, \tau) dw.$$
As $\gamma \rightarrow 0$ we obtain that $g(w, \tau)$ satisfies the Fokker-Planck equation

$$\frac{\partial g}{\partial \tau} = \frac{\lambda}{2} \frac{\partial^2}{\partial w^2} \left( D(|w|)^2 g \right) + \frac{\partial}{\partial w} \left( P(|w|)(w - m(t))g \right).$$

The balance $\gamma \rightarrow 0$ and $\sigma \rightarrow 0$ in such a way that $\sigma^2/\gamma = \lambda$, allows to recover in the limit the contributions due both to compromise propensity and the diffusion.

Other limits can be considered, which are diffusion dominated ($\sigma^2/\gamma = \infty$) or compromise dominated ($\sigma^2/\gamma = 0$).

However, the formation of an asymptotic profile for the opinion is linked to the first balance [Ben-Naim E. (2005)].
Other models

- Pure diffusion and drift equations connected to this matter have been recently introduced in [Slanina F. Lavička H. (2003)].
- These equations, in our picture, refer to diffusion dominated ($\sigma^2/\gamma = \infty$) or compromise dominated ($\sigma^2/\gamma = 0$) limits.
- The diffusion dominated limit takes into account only the second-order term into the Taylor expansion.
- Suppose

\[ \frac{\sigma^2}{\gamma^\alpha} \to \lambda, \quad \alpha < 1. \]

- Then

\[ \tau = \gamma^\alpha t, \quad g(w, \tau) = f(w, t) \]
Other models

- We obtain

\[
\frac{d}{d\tau} \int_I g(w)\phi(w) \, dw = \left\langle \frac{1}{\gamma^\alpha} \int_{I^2} [(\gamma(w_\ast - w) + \eta D(|w|))\phi'(w) \\
+ \frac{1}{2} (\gamma(w_\ast - w) + \eta D(|w|))^2 \phi''(w)]g(w)g(w_\ast) \, dw_\ast \, dw \right\rangle + R(\gamma, \sigma)
\]

- Since \( \alpha < 1 \), the first order term in the Taylor expansion vanishes in the limit, and \( g \) satisfies the diffusion equation

\[
\frac{\partial g}{\partial \tau} = \frac{\lambda}{2} \frac{\partial^2}{\partial w^2} (D(|w|)^2 g).
\]

- Set \( D(|w|) = \sqrt{1 - w^2} \), \( \lambda = 2 \). Then

\[
\frac{\partial g}{\partial \tau} = \frac{\partial^2}{\partial w^2} [(1 - w^2)g].
\]
The compromise dominated \((\sigma^2/\gamma = 0)\) limit corresponds to the scaling

\[
\frac{\sigma^2}{\gamma^\alpha} \rightarrow \lambda, \quad \alpha > 1.
\]

In this case, we scale as

\[
\tau = \gamma t, \quad g(w, \tau) = f(w, t).
\]

The diffusion part disappears in the limit and we obtain the pure drift equation

\[
\frac{\partial g}{\partial \tau} = \frac{\partial}{\partial w} (P(|w|)(w - m(t))g).
\]
Other models III

- The choice $P(|w|) = 1 - w^2$ has been considered in [Slanina F. Lavička H. (2003)].
- In this case [Aletti G., Naldi G., Toscani G. (2007)]

$$\frac{\partial g}{\partial \tau} = \frac{\partial}{\partial w} \left( (1 - w^2)(w - m(t))g \right).$$

- $m(\tau)$ satisfies

$$\frac{dm(\tau)}{d\tau} = -m(\tau) \int_{I} w^2 g(w, \tau) \, dw + \int_{I} w^3 g(w, \tau) \, dw.$$


$$\frac{\partial g}{\partial \tau} = \pm \frac{\partial}{\partial w} \left( (1 - w^2)wg \right).$$
Steady states

- Explicit steady states if $P(|w|) = 1$, which implies conservation of the average opinion.
- Set $D(|w|) = 1 - w^2$. The steady state distribution of opinion is a solution to

$$\frac{\lambda}{2} \frac{\partial}{\partial w} ((1 - w^2)^2 g) + (w - m)g = 0$$

- We obtain

$$g_\infty(w) = c_{m,\lambda} (1 - w)^{-2+m/(2\lambda)} (1 + w)^{-2-m/(2\lambda)} \exp \left\{ -\frac{1 - mw}{\lambda(1 - w^2)} \right\}.$$ 

- The constant $c_{m,\lambda}$ is such that the mass of $g_\infty$ is equal to one.
- Note that $g_\infty(\pm 1) = 0$. The solution is regular, but not symmetric unless $m = 0$. Two picks (on the right and on the left of zero).
Steady states II

- Similar result expected from the choice \( D(|w|) = 1 - |w| \).
- The steady state distribution of opinion is a solution to

\[
\frac{\lambda}{2} \frac{\partial}{\partial w} \left( (1 - |w|)^2 g \right) + (w - m)g = 0.
\]

- We obtain

\[
g_\infty(w) = c_{m,\lambda} (1 - |w|)^{-2-2/\lambda} \exp \left\{ -\frac{1 - mw/|w|}{2\lambda(1 - |w|)} \right\}.
\]

- Low regularity of \( D(|w|) \) reflects on the steady solution, which has a jump in \( w = 0 \).
- As in the first case, the presence of the exponential assures that \( g_\infty(\pm 1) = 0 \).
Steady states III

- Last, consider $D(|w|) = \sqrt{1 - w^2}$.
- The steady state distribution of opinion is a solution to

$$
\frac{\lambda}{2} \frac{\partial}{\partial w} \left((1 - w^2)g\right) + (w - m)g = 0.
$$

- We obtain

$$
g_\infty(w) = c_{m,\lambda} \left(\frac{1}{1 + w}\right)^{1-(1+m)/\lambda} \left(\frac{1}{1 - w}\right)^{1-(1-m)/\lambda}.
$$

- Since $-1 < m < 1$, $g_\infty$ is integrable on $\mathcal{I}$.
- Differently from the previous cases, however $g_\infty(w)$ tends to infinity as $w \to \pm 1$, and it has no peaks inside the interval $\mathcal{I}$.
- Extreme opinions win!
Conclusions

- Various kinetic models of opinion formation, based on binary interactions involving both compromise and diffusion properties in exchanges between individuals have been introduced.
- A suitable scaling of compromise and diffusion allows to derive Fokker-Planck equations for which it is easy in many cases to recover explicitly the stationary distribution of opinion.
- Among these Fokker-Planck equations, one is emerging and takes the role of the analogous one obtained in [J.P. Bouchaud, M. Mézard (2000), Cordier S., Pareschi L., Toscani G. (2005)] for the evolution of wealth.
- The main feature of this equation is that moments can be evaluated in closed form, and possible models of hydrodynamics can eventually follow as in classical kinetic theory of rarefied gases.